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A CRITICAL EXAMINATION OF INCLINED CRACKING EQUATIONS

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Critical Examination of Inclined Cracking Equations" submitted by Edward Milton Morrison in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

An analytical investigation was conducted to study the development and suitability of the more recent inclined cracking theories. It involved a comprehensive examination of the development of diagonal or inclined cracks and the factors affecting such cracking in reinforced concrete beams without web reinforcement.

Six different inclined cracking theories and their basic assumptions are presented, showing how the critical shear and diagonal tension stresses are computed according to each of the theories. The relative merits of each expression are then compared with observed behavior.

The empirical correlation of calculated values with the corresponding observed inclined cracking loads for 194 test beams indicated that the ACl-ASCE Committee 326 Equation gave the best general solution.

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CHAPTER I

INTRODUCTION

1.1 General Remarks

The design of reinforced concrete beams for shearing forces and diagonal tension stresses has been studied for nearly 65 years, and a considerable number of hypotheses and design recommendations have been proposed during that time. However, the advance of knowledge in this field has been slow. A study of available test data and theories regarding diagonal cracking and shear failure reveal that a detailed understanding of the fundamentals of shear cracking and failure has not been obtained. As a result there is a considerable amount of disagreement among authors and codes on the phenomenon of shear failure.

The major difficulties in developing a general solution to the problem lie in the non-homogeneity of concrete, the non-linearity of its stress-strain diagram and in the indeterminacy of the internal force system of a cracked reinforced beam. Thus the approach used in understanding the mechanism of shear failure and the concepts presented by the different investigators vary

according to the factors which they consider particulary critical. To date, both analytical and empirical treatments have been proposed to correlate these critical factors into diagonal tension theories.

During the past decade, extensive investigations were conducted on the shear strength of reinforced concrete beams. These investigations have provided a great deal of data and information on the behavior of reinforced concrete beams with respect to shear failure and diagonal cracking. The material presented in this thesis is an attempt to analyze the factors considered and the suitability of recent inclined cracking theories.

1.2 Object

This thesis represents a pilot study on shear and diagonal tension which may be used as a basis for future research. The object of this investigation was to study the development and suitability of the more recent inclined cracking theories.

In order to comprehend the various inclined cracking theories it was necessary first to examine the development of diagonal cracks and the factors affecting diagonal tension strength in reinforced concrete beams. Due to the importance

of the tensile strength of concrete in assessing the diagonal tension strength of a beam, the relationship between the compressive strength, the tensile strength and the flexure strength of concrete is reviewed in this thesis.

To study the suitability of each theory, the documented behavior of test beams is compared to the behavior assumed in each of the theories and the equations developed according to each of the theories are compared with the corresponding observed diagonal tension cracking loads of several investigations.

1.3 Scope of Investigation

This investigation included the empirical correlation of calculated values, obtained by six separate theories, with the corresponding observed diagonal tension cracking loads for 194 test beams. The test beams used in this correlation were the same beams used by ACI-ASCE Committee 326(1962) in developing their proposed design equation. The test beams consisted of four different types of rectangular beams without web reinforcement:

(1) Simple beams with one or two concentrated loads.

(2) Stub beams with one concentrated load.

(3) Restrained beams loaded symmetrically at the overhangs and with one or two concentrated loads in the center span.

(4) Two span continuous beams under different arrangements of concentrated loads.

The test results have been compared statistically to inclined cracking load equations presented or developed by:

(1) 1956 ACl Code (1956)

(2) Guralnick (1960)

(3) ACl-ASCE Committee 326 (1962)

(4) Van den Berg (1962)

(5) Kani (1964)

(6) Sozen and Hawkins (1962)

Several other recently published theories and tests have been examined and provide supplementary information on the influence of factors affecting diagonal tension beam behavior.

1.4 Symbols

All the symbols are explained when they are first introduced. They are collected here for ready reference. Certain subscripts are applied to many symbols and refer to the major symbol at the subscript condition.

Subscripts:

X_{cr} = Condition X at inclined or flexural cracking

X_{ULT} = Condition X at ultimate

X_c = Condition X for concrete

X_s = Condition X for steel reinforcement

Cross-sectional Constants

A_g = Gross area of cross section

A_s = Total area of longitudinal reinforcement

I = Moment of inertia of the gross concrete cross section about the centroidal axis.

Q = Statical moment about the neutral axis of that portion of cross section lying beyond a line through the point in question and parallel to the neutral axis.

b = Width of rectangular beam or width of flange for flanged beams.

b' = Thickness of web for flanged beam.

d = Effective depth of the longitudinal tension reinforcement.

h = Total depth of rectangular beam.

a = Shear span = distance from concentrated load to reaction in a beam loaded with one or two concentrated loads = M/V.

c = Depth of cover for reinforcement.

ΔX = Crack spacing at level of main reinforcement.

S = Maximum crack height measured above main reinforcement.

$\sum_0$ = Sum of reinforcing bar perimeters.

L = Span length.

<u>Dimensionless Factors</u>

p = As/bd = Reinforcement ratio.

a/d = Ratio of shear span length to effective depth.

K and C = Empirical constants.

j and j'= Empirical internal lever arm coefficients.

$\Delta X/S$ = Crack factor

η_d = A specific coefficient relating cross-sectional shape of reinforcing bars.

Stresses

Concrete

f'_c = compressive strength determined from 6 by 12 in. control cylinders.

f'_t = Assumed tensile strength of concrete.

f_r = Assumed modulus of rupture.

E_c = Assumed modulus of elasticity of concrete.

v = Unit or nominal shear stress.

$f_{D.T.}$ = Principal tension stress.

f_t = Flexural tension stress.

v'_c = Assumed shear stress corresponding to the strength of concrete subjected to pure shear.

ε_c = Assumed concrete strain.

Steel

E_s = Assumed modulus of elasticity of steel.

ε_s = Measured steel strain.

f_y = Yield stress for reinforcement.

Loads

P = Applied concentrated load at any stage of loading.

M = Bending moment at any stage of loading. For beams loaded with one or two concentrated loads

M is the moment at the concentrated load or at the reaction.

V = Total applied shear at a section.

T = Tension force in the longitudinal reinforcement.

C' = Compressive force in the concrete.

CHAPTER II

REVIEW OF OBSERVED INCLINED CRACKING BEHAVIOR

2.1 Introductory Remarks

In order to discuss the various inclined cracking theories it is necessary first to consider the development of inclined or "shear" cracks in reinforced concrete beams. Although the formation of such cracks in reinforced concrete beams is highly variable, investigators have observed several common characteristics by which the crack patterns can be distinguished and grouped.

The behavior of reinforced concrete beams is essentially elastic until cracking. During this initial stage of loading deflections are proportional to the loads. The elastic stage ends with the formation of either flexural or inclined cracks.

Flexural cracks which result from horizontal tensile stresses will be considered only in relation to their effect on "shear" or inclined cracking. This limitation is imposed because bending stresses predominate in flexural cracking. If

sectional properties are such that diagonal tension is not a controlling influence, the flexural cracking eventually results in a flexural failure at a section of maximum moment. This latter type of failure may take one of two forms: the steel may yield at the critical crack, or, alternately the concrete section is so reduced by the vertical rise of the crack that crushing of the compressive concrete occurs.

Inclined cracking, which may produce a form of shear failure, is caused by a combination of bending and shearing stresses. Beam action is disrupted by inclined cracks because the principal tensions in the web of the beam cannot be transferred across the crack. The resulting redistribution of internal stresses, in turn results in ultimate loads and deflections smaller than those corresponding to a flexural failure.

2.2 Mechanisms of Inclined Cracking

(a) Shear-Proper

For loads very close to the support, (a/h 1), as shown in Figure 2.1 the development of inclined cracks is greatly affected by the presence of the nearby load and reaction. Laupa, Siess and Newmark (1955) refer to this type of failure

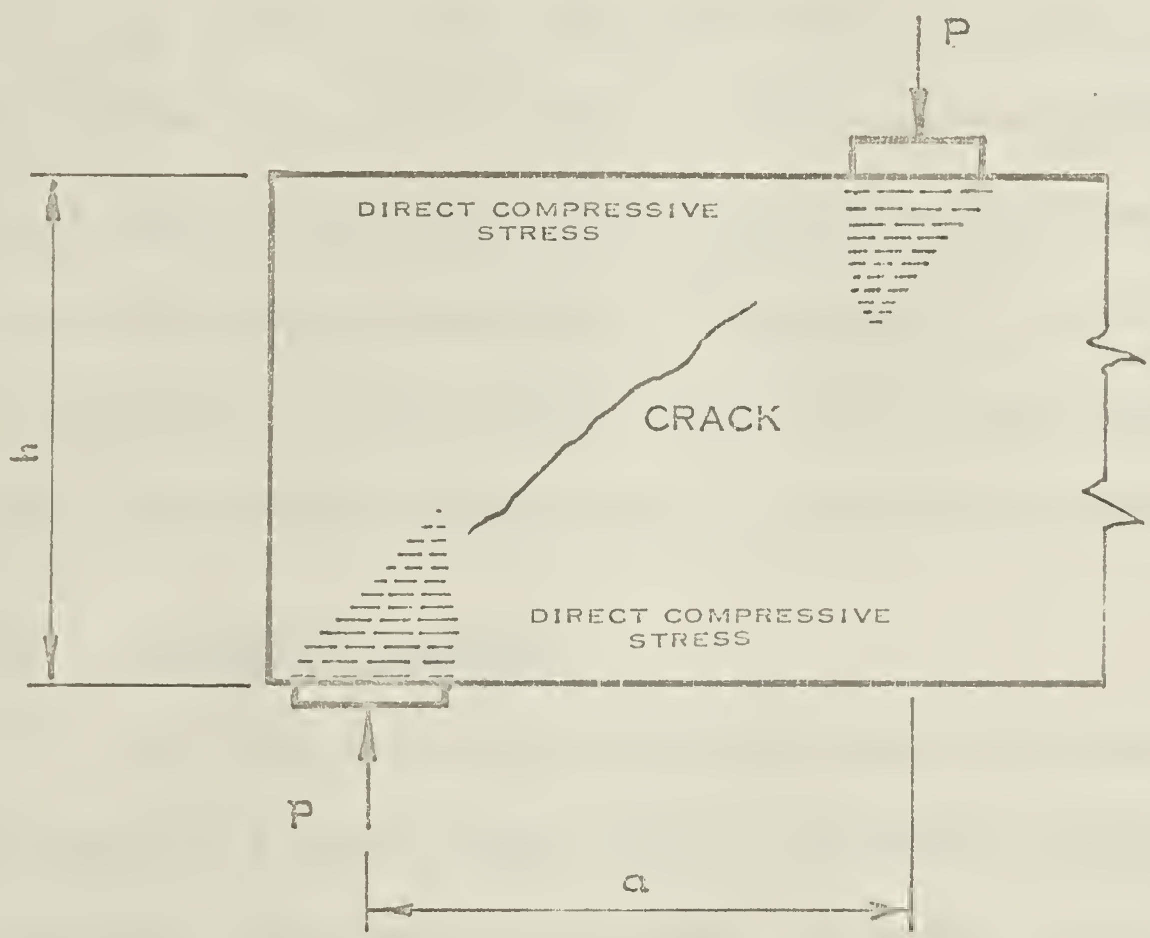


FIGURE 2.1 DIAGONAL CRACKING - SHEAR PROPER

mechanism as "Shear-Proper" and discuss tests of members loaded in this way. Because the shear stress, v, is large compared to the flexural tension, f_t, the vertical compressive stresses in the vicinity of the reaction and load points reduce the magnitude of the principal tension stresses, thus increasing the inclined cracking load and the inclination of the crack with the horizontal axis of the beam. The relative importance of the local compression stresses increases as the distance between load and support decreases. This type of cracking will not be discussed in this paper since it is restricted to very short shear spans which seldom occur in practical beams.

(b) Inclined Cracking

In this thesis an inclined crack or shear crack has been defined as a crack formed due to principal tensile stresses resulting from combined bending and shear. More than one inclined crack may form in any one beam before ultimate load is reached. The crack which forms first will be called the "initial inclined crack" and the corresponding load will be called the "initial inclined cracking load". Since the behavior of a beam failing in shear is strongly affected by the dimensions of the beam, the discussion of beam behavior will be broken into two parts based on beam dimensions.

(i) Beams with Large Ratio of Shear Span to Depth

Several investigators have suggested that inclined cracking can be analyzed by considering its development in stages. The rudimental formation of cracks under increasing load for a beam with a medium to large ratio of shear span to depth is shown in Figure 2.2. Stage "A" represents vertical flexural cracking in a region of maximum moment with stage "B" showing the development of similar cracks in the shear span. The stages "D" and "E" sometimes occurred simultaneously, while in other tests further increments of load beyond that producing stage "D" were required to bring about collapse. These latter increments of load were found to be relatively small.

The stages in the development of an inclined crack are shown in more detail in Figure 2.3. Flexural cracks form in the region of maximum moment and rise vertically above the soffit, (Stage (1)). As the flexural cracks extend higher under increased load, those cracks starting in the shear span tend to bend over slightly towards the load points, (Stage (2)). One or two of these flexural cracks will generally trigger an

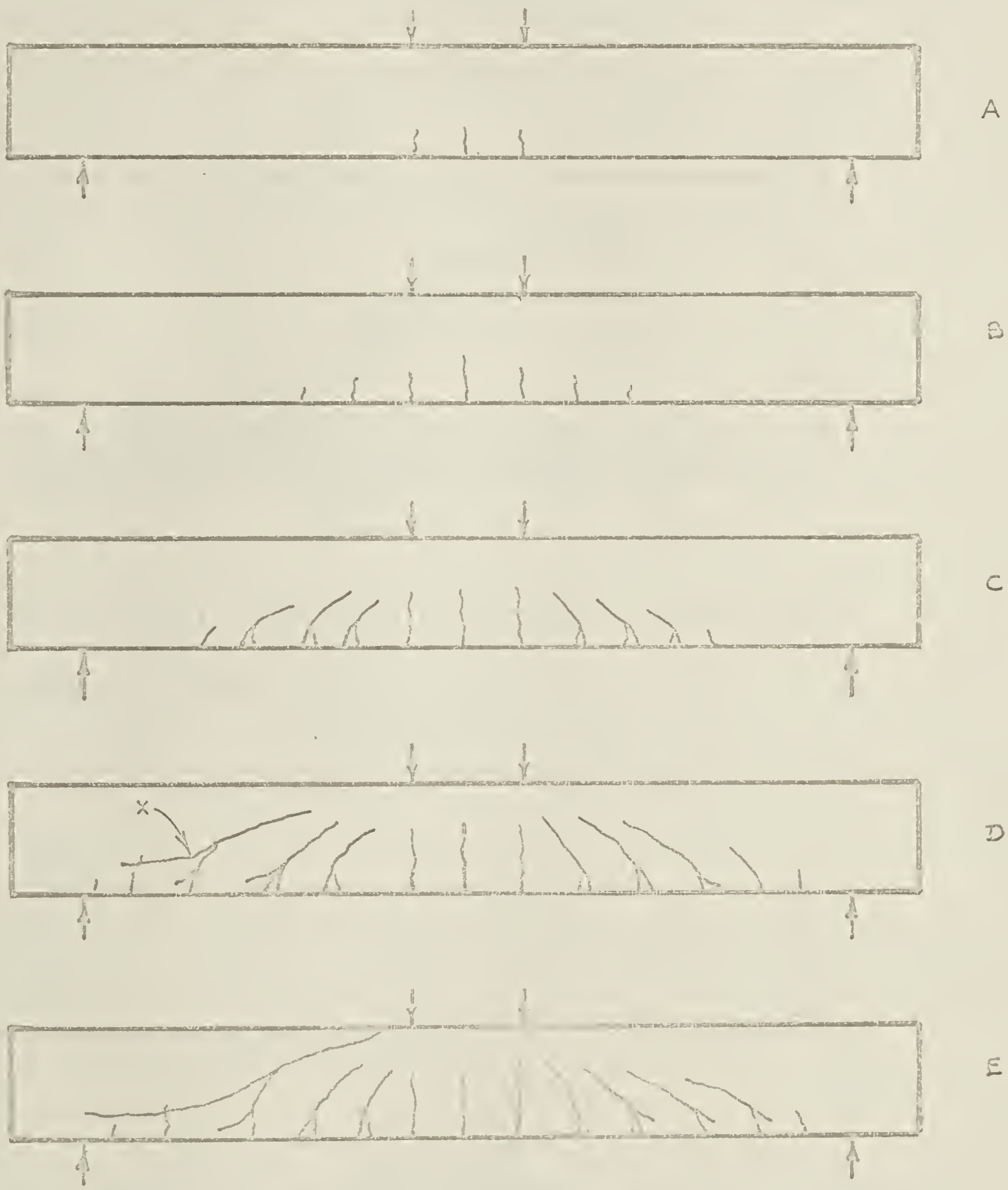


FIGURE 2.2 THE FORMATION OF INCLINED CRACKS UNDER INCRASING LOAD

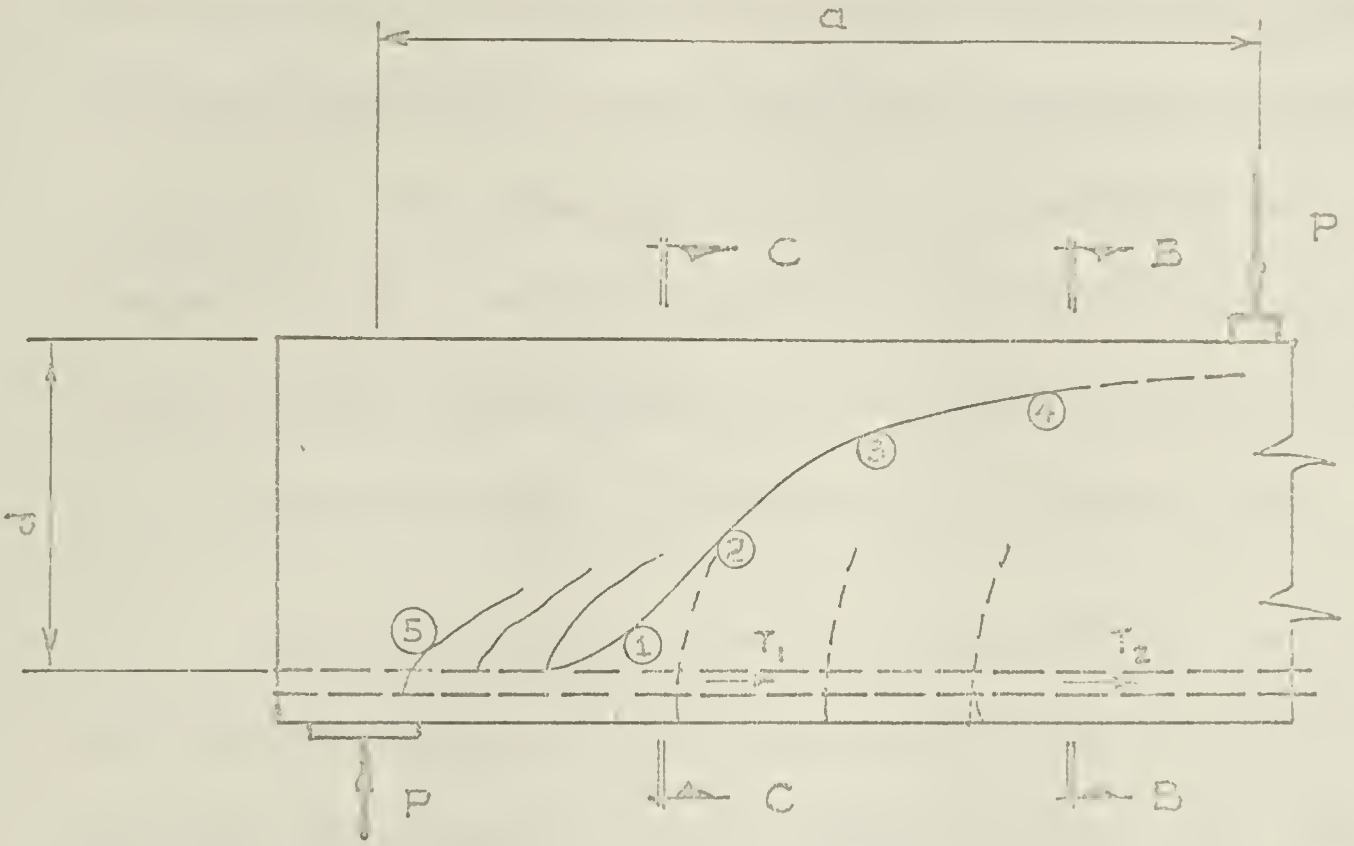


FIGURE 2.4 INCLINED CRACKING FOR BEAM WITH MEDIUM TO SMALL RATIO OF a/d (1 5 a/d 3.5)

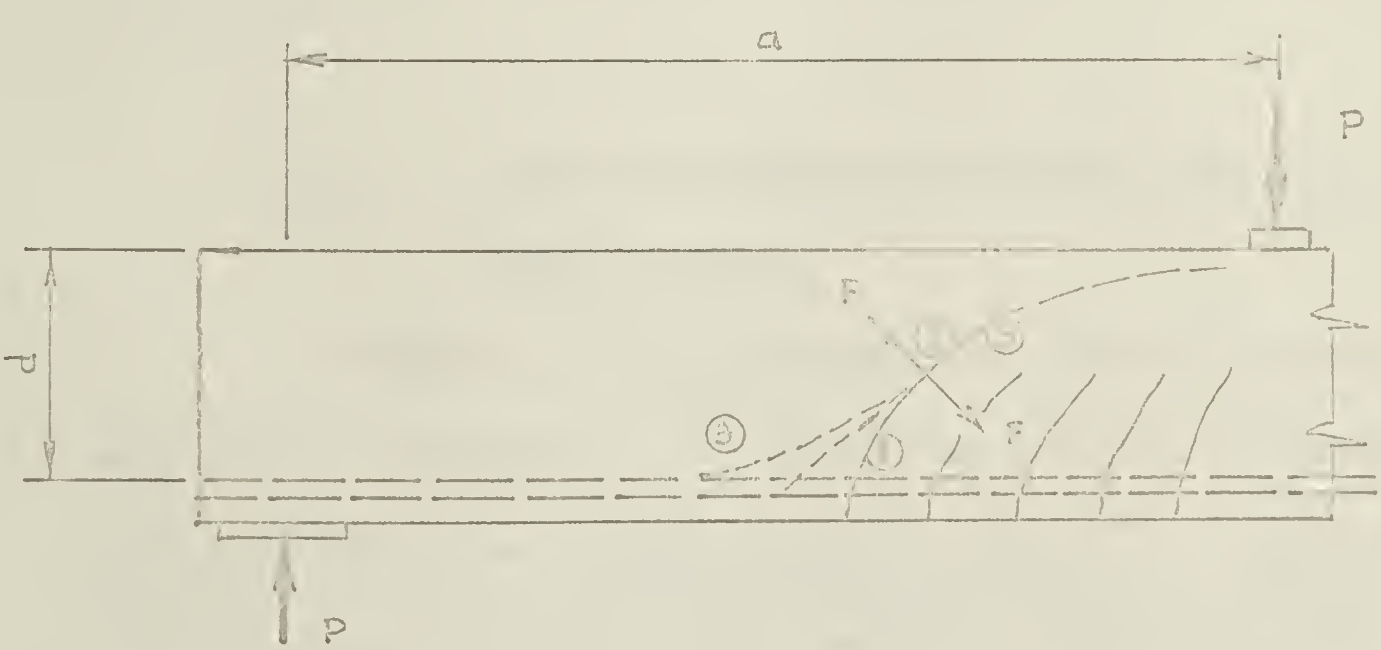


FIGURE 2.3 INCLINED CRACKING FOR BEAM WITH LARGE RATIO OF a/d (3.5 a/d = 6 0)

inclined crack (Stage (3) in Figure 2.3) in the region of maximum shear due to the increased stresses at the upper end of the crack. The diagonal crack often develops as a growth or extension of a vertical moment crack which turns into an inclined crack in the neighborhood of the neutral axis as shown by (3) or it forms suddenly as shown by the dashed line.

For large shear span to depth ratios, in beams without web reinforcement, failure generally occurs immediately following the development of the inclined crack when this crack extends through the compression zone, or when this crack extends along the reinforcement. Generally, these modes of failure can be overcome by the use of the proper amount of stirrups.

(ii) Beams With Medium to Small Ratios of Shear Span to Depth

In beams in this category the diagonal tension cracks are often initiated at or above the tension reinforcement and are inclined from their inception (Stage (1) in Figure 2.4). In this case, however, the diagonal crack does not proceed immediately to failure as in the case for beams with long shear

spans. Instead, the crack encounters resistance as it moves up into the zone of compression, becomes more horizontal and stops at some point such as (2) in Figure 2.4.

With further load, the tension crack extends to (3) at a fairly flat slope until finally sudden failure occurs at (4) due to compression failure of the concrete over the head of the crack.

In almost every case the crack will be associated in some way with a moment or flexural crack (shown by the vertical dash line in Figure 2.4). However, the inclined crack may occur near the flexural crack, above it or may cross it. In every case the inclined crack will tend to crack back down to point (1) before reaching failure at (4).

Redistribution of internal stresses brought about by the loss of the principal tensions in the web is the main reason for the final stages of development in the inclined crack. Hence, at the same time that cracking is developing in the compression zone, a serious concentration of bond stress develops at (1). Before cracking, the tension T_1 in the

reinforcing steel at C is less than the tension T_2 at Section B. As the crack develops and moves upward and towards the load point; the tension, T_1 at C increases due to loss of beam action between C and B. Thus, high bond stresses are imposed on the section to the left of the crack.

The larger bond stress required in the left portion of the beam (Figure 2.4) leads to a larger unit shear stress just above the steel. This is one factor leading to the localized diagonal cracking indicated by (5). This cracking is accentuated by some vertical load carried across the crack by the reinforcing bars acting as dowels. The vertical tension due to dowelling aids in splitting and so reduces the bond in the bars, possibly leading to a bond type failure.

A reinforced concrete beam having a medium a/d ratio and failing as a result of diagonal tension will generally fail in one of the following ways.

(1) Destruction of compression zone at the upper end of an inclined crack.

(2) Bond failure.

(3) Splitting failure.

In each case the beam action is disrupted or destroyed by the inclined crack with the result that the failure load and deflection are both smaller than anticipated if the beam had failed in flexure.

CHAPTER III

FACTORS AFFECTING DIAGONAL TENSION STRENGTH

3.1 Introduction:

A study of a number of published theories about the formation of inclined cracks* showed that most investigators have attempted to empirically correlate most or at least some of the factors affecting shear in order to produce relationships for the observed strength. In order to better comprehend the many factors, affecting the inclined cracking load and shear strength, it is useful to discuss them separately and review the influence of each variable. The following factors will be considered in this review:

1) The ratio of the length of the shear span to the depth, a/d and moment to shear ratio, M/Vd.

2) Concrete properties.

* Notably by, Clark (1951), Elstner, Moody, Viest, Hognestad (1954) Sozen, Zwoyer, Siess (1953), Ferguson (1956), Guralnick (1960), Van den Berg (1962), ACI-ASCE Committee 326 (1962) and Kani (1964).

3) Percentage of tension reinforcement.

4) Member size, shape and b/d ratio.

5) Type of loading.

6) Other considerations.

These parameters are summarized in Table 3.1, at the end of the chapter for subsequent reference.

3.2 Influence of "Shear Span to Depth Ratio"

The relative length of a beam was pointed out to be a major variable as early as 1909 by Talbot but was never expressed in mathematical terms until Clark (1951) introduced an expression involving distance "a" from the load to the support of a beam loaded with one or two concentrated loads, and the effective depth of the beam, "d". The length "a" is commonly referred to as the "shear span". Moody (1956) observed that the shear span length, "a", was equal to the moment to shear ratio, M/V. The development of the M/Vd ratio concept provided a more rational factor as it has physical significance at any cross-section of a beam whereas the shear span "a" has no meaning for uniform loading.

The significance of either M/Vd or a/d in an equation

appears to be that it relates the effect of horizontal flexural tension on the diagonal tension stresses. Investigations such as those by Kani (1964) and Morrow and Viest (1957) have indicated that the shear failure moment capacity of simple span beams loaded with concentrated loads can be related to the full flexural moment capacity as a function of the a/d ratio. Figure 3.1 which is based on figures presented by Morrow and Viest and by Kani, illustrates the effect of the a/d ratio on the moment capacity of a beam with a given cross-section and steel percentage. Similar figures have been presented by other investigators. Figure 3.1 is plotted for beams without web reinforcement.

In Figure 3.1 four different regions of a/d ratio are shown. Within any given region the beam characteristics are:

REGION I - The full flexural capacity of the beam is developed for a/d values greater than approximately 5.5. The exact value of a/d is a function of several other variables such as concrete strength and steel percentage. In Region I the inclined cracking moment is greater than the flexural failure moment.

REGION II - Inclined cracking occurs and the beam fails almost immediately in shear with an ultimate moment less than

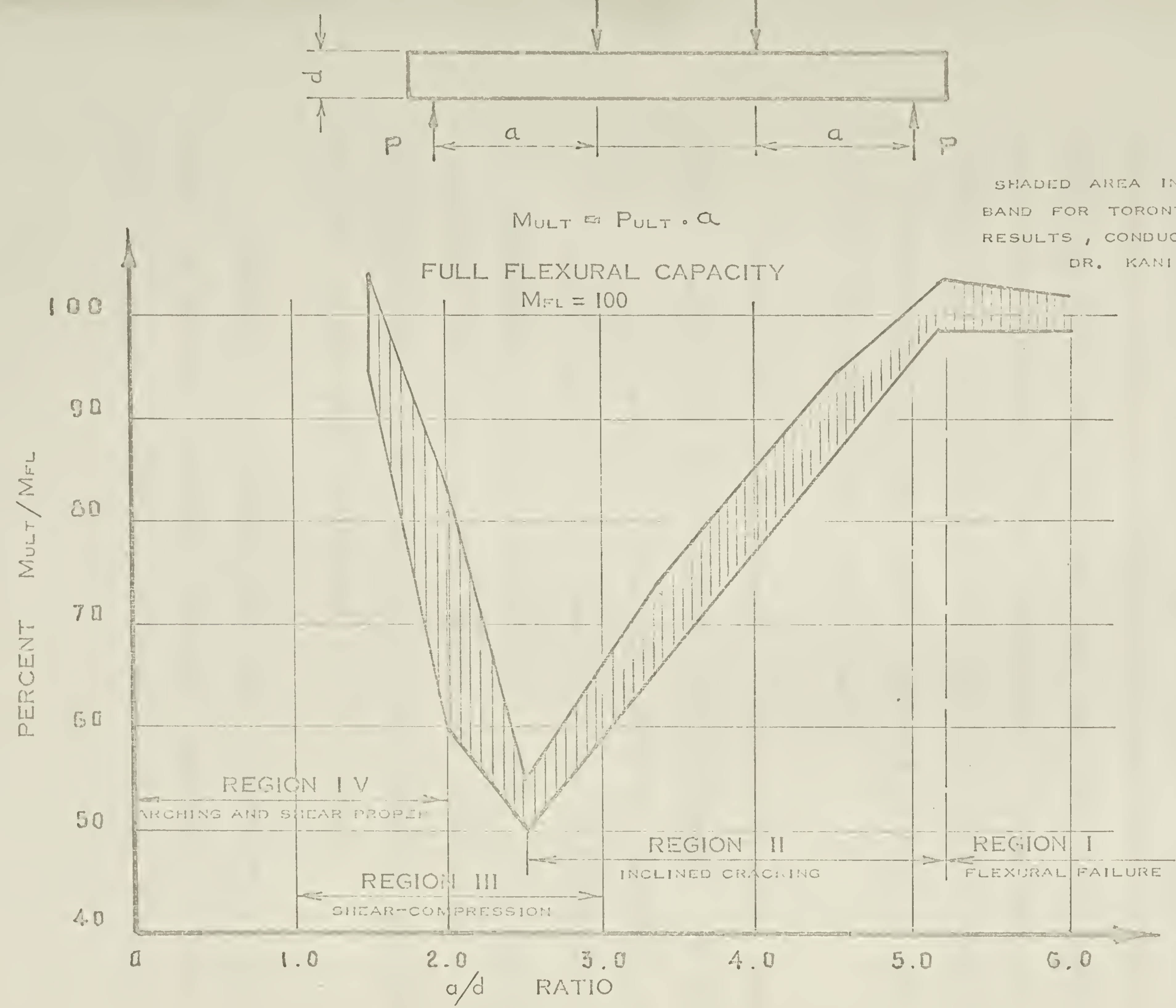


FIGURE 3.1 BEAM CAPACITY vs a/d RATIO

the full flexural capacity. The ratio of the shear failure moment to the inclined cracking moment is close to one and decreases to one as the a/d ratio increases from 2.5 to 5.5. Morrow and Viest have suggested that this type of failure occurs because the capacity of the cracked beam is less than the inclined cracking load. Most investigators indicate that in this range the inclined cracking load can be regarded as the ultimate load for beams without web reinforcement.

REGION III - In this range of a/d the beam can carry a moment larger than the inclined cracking moment. In this region the inclined crack forms and, after the addition of up to 100 percent more load the beam fails due to crushing of the compression zone over the inclined crack. This type of failure was first described by Zwoyer and Siess (1954). If the reinforcement is well anchored, shear capacity generally increases as a/d ratio decreased in this region.

REGION IV - Considered the most complex region with respect to inclined cracking and shear capacity, as beam behavior suggests a combination of direct shear, flexure,

arch action and thrust. In general, this is the region of "shear proper" failures and will not be discussed in this thesis.

It is apparent from the above discussion and the descriptions of cracking behavior in Chapter II that the a/d ratio, or the M/Vd ratio in the case of uniform load, is a significant parameter.

3.3 Influence of Concrete Properties

Beginning with the earliest design specifications, the calculation of the shear strength of reinforced concrete members has depended upon concrete quality, generally expressed by the compressive strength of concrete, f'_c. However, because the inclined cracking of reinforced concrete is considered a tension stress phenomenon it is important to also consider the tensile or flexure strength of concrete.

Sozen (1959), Guralnick (1960), Van den Berg (1962), Kani (1964) and others have concluded that the inclined cracking load is more closely related to the tensile strength than to the compressive strength of concrete. Indeed,

Van den Berg found a linear relationship between the cracking stress and the tensile strength of concrete. It has also been pointed out that much of the scatter in test results can be attributed to the rather wide variation of concrete properties, especially the variation of concrete tensile strengths. For further discussion on the properties of concrete, reference is made to Chapter IV, "Properties of Concrete and Steel".

3.4 Percentage of Longitudinal Tension Reinforcement

It has long been recognized that shear strengths corresponding to both diagonal tension cracking and ultimate are functions of the amount of reinforcement. However, the method of expressing the percentage of longitudinal reinforcement as a variable differs greatly from investigator to investigator, because of the numerous ways in which steel percentage affects beam behavior. The following effects of longitudinal reinforcement have been noted by investigators:

(a) Effect of steel stress and tensile properties, Mathey and Watstein (1963) found that for beams having the same a/d ratio, a linear relationship existed between the maximum computed steel stress in the shear span and both the observed

inclined tension cracking load and the shear failure load. It was further stated that a reduction in the ratio of reinforcement in comparable beams resulted in higher steel stresses and lower shear strengths. They also observed that for beams with the same steel percentage the inclined cracking load was independent of the tensile properties provided the reinforcement remained essentially elastic.

(b) Effect of Dowel Resistance. The shear resistance afforded by the reinforcement by what is called "Dowel" action has been observed to be considerable after cracking, particularly for beams with short spans. Therefore the effects of dowel action to be discussed are applicable only to a cracked section.

Krefeld and Thurston (1962) have considered the effect of dowel action and presented test data from which they evaluated the proportions of total shear carried by the dowel resistance. The dowel resistance was found to be of the order of 30 percent of the total external shear. Jones (1956), Moe (1962), Watstein and Mathey (1958) also concluded that the shear resistance afforded by the longitudinal reinforcement due to dowel action was substantial after extensive cracking in the shear span.

The majority of investigators, however, consider first diagonal cracking as failure, particularly for beams with long shear spans, repeated or long time loading, and thus disregard dowel resistance to shear.

In summary it would appear that the shearing force carried by the dowel action of the reinforcement may be significant after first diagonal cracking in shear compression type failures but in most analyses it has been considered as negligible at first diagonal cracking.

(c) Effect of Cracking. Chapter II indicated how the extent and distribution of flexural cracking influenced inclined cracking. Because flexural cracking plays an important role in the mechanism of inclined cracking, the relationship between flexural cracking and longitudinal steel is important.

Figure 3.2, derived in Appendix A, shows the relationship between moment and the height of a flexural crack in a region of pure flexure, as a function of steel percentage and concrete strength. This figure shows that for a given concrete strength, an increased percentage of longitudinal reinforcement, p, increases

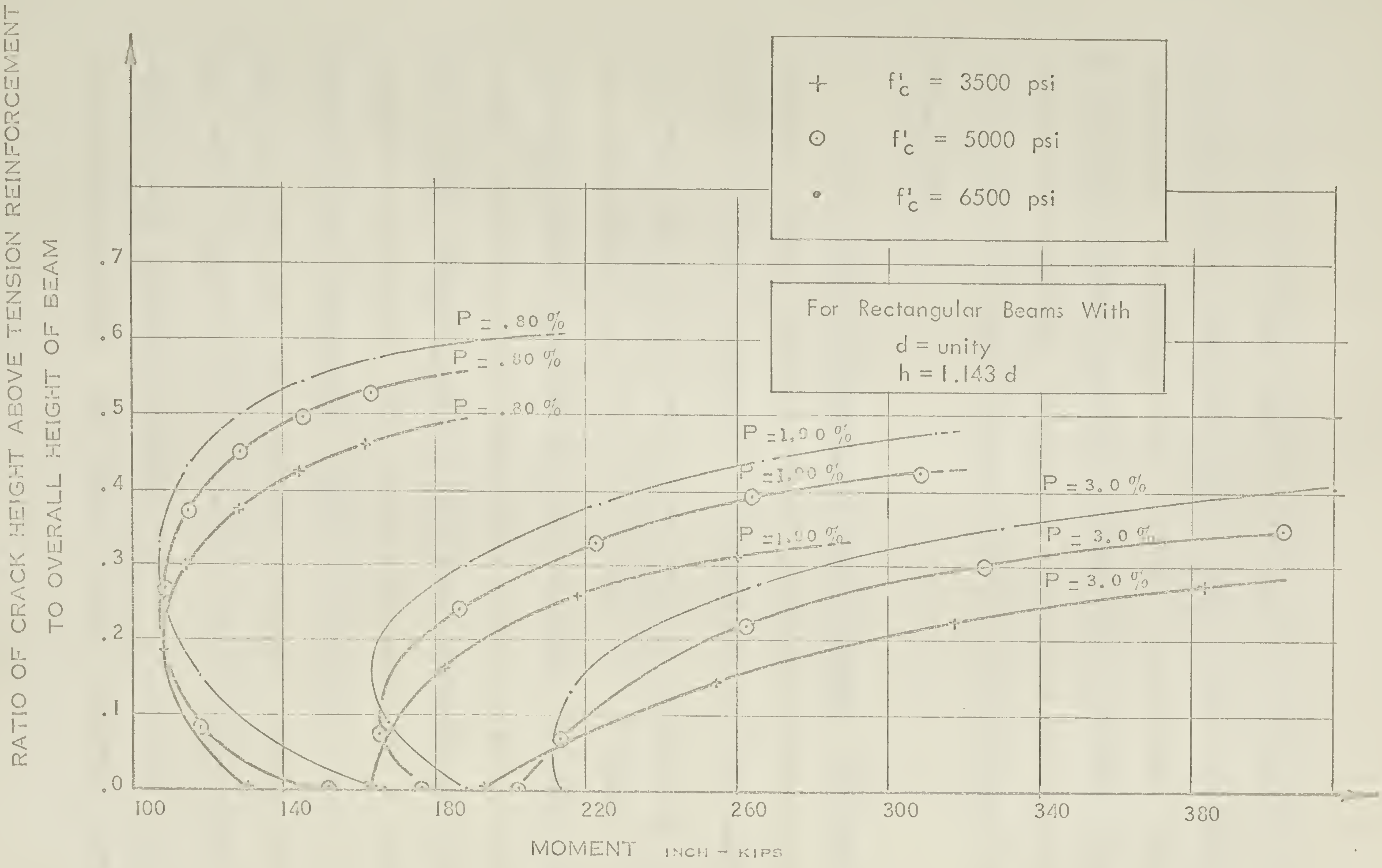


FIGURE 3.2 MOMENT CRACK - HEIGHT RELATIONS

the flexural cracking moment slightly and considerably decreases the height of the flexural crack. Thus, other conditions remaining similar, it appears that the effect of flexural cracking on inclined cracking should decrease with increased longitudinal reinforcement because cracking is restrained and grows more slowly and as a result disrupts the principal stresses to a minimum degree.

(d) Effect of Bond and End Anchorage. The ability of a beam to sustain load from first inclined cracking until failure in shear is directly influenced by the bond and end anchorage capacity of the tension steel, particularly for beams with medium to small shear spans. The formation of inclined cracks, combined with the breakdown in bond transfer between flexural cracks changes the stress relationships of the beam. Such redistribution of internal stresses was considered briefly in Chapter II and is discussed more fully by Moody, Viest, Elstner, Hognestad, (1954) and Ferguson (1956). Thus, shear capacity of a beam will be effectively reduced if the bond and end anchorages are insufficient to maintain the "beam action" until either a diagonal tension failure or flexural failure has occurred. This reduced shear capacity appears

to agree with accepted bond theory, that is poor bond resistance reduces the number but increases the width and height of cracks and tends to increase the rotation of the relative cracked sections. Moreover, as was pointed out by Menzel and Woods (1952), these effects of deficient bond are undesirable as they raise the neutral axis and tend to produce horizontal splitting and cracking in the anchorage ends.

In recent discussions to a paper by Kani (1964) two opposing thoughts with respect to the interaction of bond on shear capacity are expressed. The majority of discussers indicated that improvements in the shear capacity of concrete beams can be realized through improved bond. Kani, however, stated that the better the bond the lower the shear capacity. There is confusion here simply because bond appears to have one effect on inclined cracking and another on shear compression failures. Thus, according to Kani (1964), better bond reduces strength of beams failing at the inclined cracking load but according to nearly everybody else better bond increases the shear-compression load. Both conclusions are acceptable since they deal with different ranges of a/d.

3.5 Effect of Member Size and Shape*

Very little data is available on the effect of overall beam dimensions on shear strength of rectangular members without web reinforcement. To date most of the tests have been concerned with members having width to depth ratios in the neighborhood of 0.5 with only limited investigation into the affect of member size. De Cossio (1964) in a discussion of the Report of ACl-ASCE Committee 326, presented data from which he concluded that:

(1) As the ratio of width to depth increases, there is an increase in shear strength.

(2) The size, measured by the effective area bd, had no significant effect on the inclined tension cracking strength.

A limited number of tests by Krefeld and Thurston (1962) were inconsistent with De Cossio's second conclusion, however. In their tests, Krefeld and Thurston found that increased size,

* The effect of size is represented by bd or shear intensity V/bh, and shape is represented by the ratio b/d.

measured by shear intensity, V/bh, produced lower relative shear intensities at failure.

Tests conducted by Sozen, Zwoyer and Siess (1959) and by MacGregor (1960) on "I-shaped" prestressed beams showed that as the web thickness was reduced the type of cracking and failure changed. Similarly, from tests on T-Beams without stirrups, Ferguson and Thompson (1953) concluded that extra web area is helpful even where it does not increase the minimum "b" width. This effect is studied more fully in a subsequent chapter and attention is called only to the fact that the inclined cracking and ultimate shear capacities may change with beam section shape.

3.6 Type of Loading

Various types of beam loading are discussed below. The effect of axial load on inclined cracking and shear strength of frame members is also briefly examined.

As failure depends on the state of stress, the manner in which the loads are applied to the beam will influence its load carrying capacity. Ferguson (1956), for example, conducted

tests to compare the capacities of beams loaded under so called "point loads" producing compressive forces in the top fibre of the beam, and beams under framed conditions where vertical shears are produced along the sides of the beam. He discovered that beam shear capacity could be decreased when load was applied as external shears as in the latter case. Ferguson concluded that the "restrain factor must influence the ability of the intact concrete section to sustain load by retarding the penetration of the diagonal crack for shear span ratios of a/d, up to 3.0.

Although the effect of uniformly distributed loads has not been properly evaluated to date, Moody et al (1956) and the ACl-ASCE Committee 326 (1962) have interpreted the available tests to indicate that the influence of the M/Vd ratio on beam behavior for uniformly distributed loads is quite similar to that for symmetrical two point loads. The shear capacity of uniformly loaded beams was higher than that obtained from comparable beams with concentrated loads. This increased beam shear capacity was explained by the fact that the uniformly distributed load changes the relation between shear and moment

along the beam length. Thus, in a beam loaded with concentrated loads the maximum shear and moment occur at the load point while in a uniformly loaded beam the moment is less than maximum in regions of high shear.

Tests by Van den Berg (1962) suggest that the inclined cracking load is a function of the rate of change of moment (i.e. shear) in the shear span, rather than a function of the total magnitude of the moment.

The relationship between shearing strength and axial load was investigated by Morrow and Viest (1957), Baldwin and Viest (1958), and Hognestad (1957). The observed diagonal tension cracking loads were found to increase with increased axial load. The laboratory tests of Hognestad indicated that the shearing strength decreased very rapidly and that failures could occur at very low nominal shear stress values for girders subjected to shear, flexure and axial tension.

3.7 Other Considerations

To date, very little conclusive data is available regarding the effect of such indeterminant factors as shrinkage, creep, sustained loads and repeated loads on diagonal cracking and ultimate shear capacity.

Jones (1956) conducted limited tests whereby he concluded, with reservations, that shrinkage stresses had a substantial effect on the load at which diagonal tension cracks appeared. Furthermore, he stated, that the early appearance of these cracks tended to cause a fairly rapid extension of cracks into the compression zone and resulted in failures at lower loads than might have occurred if shrinkage stresses had been eliminated.

Krefeld and Thurston (1962) remarked that in repeated loading where inclined cracking had occurred, the recovery cycle produced redistribution of stresses which caused further disintegration of concrete. Earlier tests by Chang and Kesler (1958) indicated that 100,000 cycles of repeated loading reduced the inclined cracking load to 68 percent of the static value.

Attention is called to these observations to suggest the need for further study, Bresler (1960) stated that the resulting internal stresses produced by such factors as shrinkage, temperature variation, and various internal restraints may be just as important in determining the development of diagonal cracking as the system of external forces. In summary determination of diagonal crack initiation and propagation

requires precise knowledge of stress distribution in structural elements considering stress concentrations at the ends of cracks and the residual stresses which may be induced by bond slip, shrinkage, temperature and other volume changes. It also requires knowledge of fracture criteria for concrete which appear to depend on the state of stress, the duration and time rate of loading, and the mechanical properties of the component materials in the structure.

TABLE 3.1

FACTORS AFFECTING SHEAR CAPACITY OF REINFORCED CONCRETE BEAMS

Parameter	General Influence on Shear
1. a/d Ratio or M/Vd Ratio	- Strongly affects shear by changing the type of behavior (see Section 3.2 and Figure 3.1.) - a/d ratios of 2.5 to 3.0 give least relative shear strength.
2. Concrete Properties	- the inclined cracking level is a direct function of the tensile strength, f'_t.
3. Percent Tensile Reinforcement	
(a) Tensile strength	- the inclined cracking load is independent of the tensile strength for a constant steel percentage, if reinforcement remains elastic.
(b) dowel resistance	- may allow increase in shear capacity after cracking.
(c) cracking	- an increase in the steel percentage, p, increases the flexural cracking load and decreases the flexural crack height.
(d) bond	- better bond increases shear and compression failure shear but may reduce cracking load.
(e) anchorage	- better end anchorage produces higher ultimate shear capacity.
4. Beam Size and Cross-Section	- increase in section increases shear capacity but appears to reduce the relative shear stress: - increase in the width to depth ratio appears to increase shear strength. - reduction in the web thickness of an I-beam affects the type of inclined cracking and failure and can reduce the inclined cracking and the shear failure load.
5. Method of Loading	- vertical compression under loads applied at top of beam and above vertical reactions at the supports increases shear capacity. - axial compression increases shear capacity whereas axial tension decreases shear capacity rapidly.
6. Creep, Shrinkage and Repeated Loading	- influence not fully established but indications are that they significantly decrease shear capacity.

CHAPTER IV

PROPERTIES OF CONCRETE AND STEEL

4.1 Introductory Remarks:

As was outlined in Chapter I, the main object of this thesis is to discuss a number of inclined cracking theories and to compare them to a series of representative test results. Since the inclined cracking theories were based on certain material properties and other quantities not given in the reference data, it was sometimes necessary to assume values for the unknown quantities in making comparisons. This chapter outlines the assumptions made. Since most of the unknowns not given in the reference data were properties of the concrete such as the flexural or tensile strength, the following discussion deals mainly with concrete properties.

4.2 Properties of Concrete

The properties of concrete are a function of the age of the concrete and the ambient conditions. Absolute measures

are significant only in so far as they indicate potential qualities and this is why, in order to be of value, tests on concrete for determination of properties have to be performed under known conditions. The most common of all tests on concrete is the compressive strength test partly because it appears to be a good index of most of the other concrete properties; but mainly because most concrete structures are designed under an assumption that concrete resists compressive stresses but not tensile, hence allowable stresses are prescribed by codes in terms of the compressive strength. A further consideration is that compression tests are relatively easy to make.

(a) Stress-Strain Curve

The stress-strain relationship for concrete is determined conventionally from 6 X 12 inch test cylinders loaded in axial compression. Typical compression stress-strain curves are given by Hognestad, Hanson and McHenry (1955), Neville (1963) or Ramaley and McHendry (1947). The relationship under compression is approximately linear for the initial stages of stress and strain but soon the strain increases at a progressively greater rate until finally a limiting useful value of strain is reached, at which time failure occurs. The stress-strain curve for low strength

concrete has a long and relatively flat inelastic region, whereas with higher strength concrete the peak of the stress-strain curve is sharper with less ultimate strain.

Since the concrete compressive strengths of the beam specimens varied from 900 psi to 6800 psi, it was necessary to approximate the concrete stress-strain relationship. The assumed stress-strain relationship is illustrated in Figure 4.1.

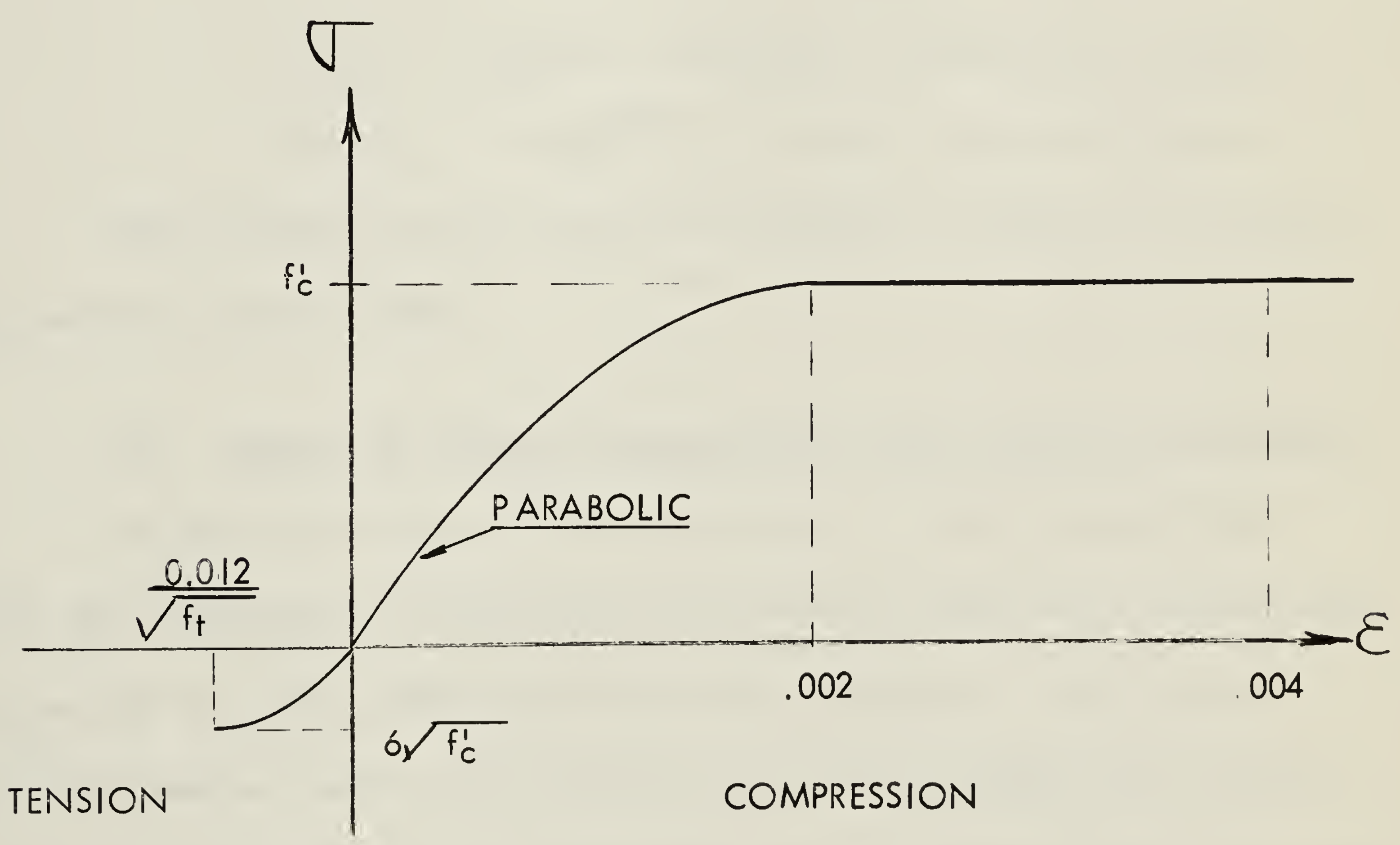


FIGURE 4.1 CONCRETE STRESS-STRAIN CURVE

Where the stress-strain curve is defined by:

(1) The initial tangent modulus:

$$E_o = 1000\ f'_c$$

(2) The maximum concrete stress at the extreme fibre:

f'_c for compression stress

$f'_t = 6\sqrt{f'_c}$ for tension stress

(3) Limiting concrete strains:

ε_o = .002" - transition from parabolic to horizontal portions of compression stress-strain curve.

ε_{ult} = .004" - ultimate compressive strain

$\varepsilon_{ult_T} = .012/\sqrt{f'_c}$ - ultimate tensile strain

This stress-strain curve is somewhat similar to the one assumed by Parme (1962).

(b) <u>Relation Between Compressive and Tensile Strengths</u>

At the beginning of this section, it was pointed out that the compressive strength was commonly used as a convenient yardstick for the other qualities of concrete. The results of compression tests can possibly be used as qualitive indications of other important qualities; but in view of the numerous factors influencing the ratio of the strengths no simple relation is generally applicable. Figure 4.2 taken from Neville

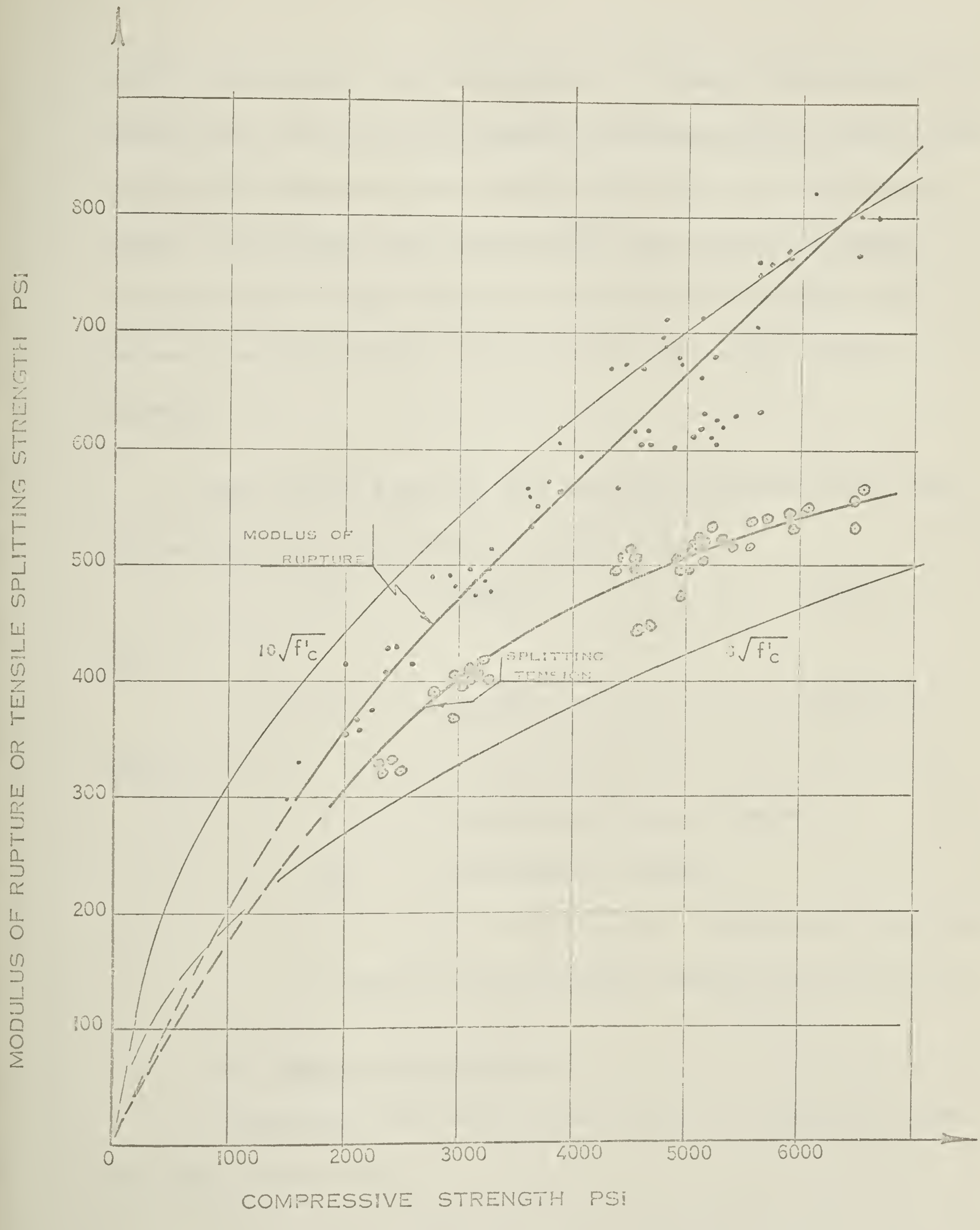


FIGURE 4.2 CONCRETE STRENGTH RELATIONSHIPS

(1963) illustrates that while average values of the relations between the strength properties of concrete can be established, individual values of these ratios may vary over a very wide range. This figure also shows that there is not a linear relationship between compressive and tensile strength, nor between the modulus of rupture and the splitting tension strength.

The tensile strength and modulus of rupture have been defined by empirical formulae of the type:

$$f'_t = C\sqrt{f'_c} \tag{4.1}$$

And:

$$f_r = C_1\sqrt{f'_c} \tag{4.2}$$

Where:

f'_t = Tensile splitting strength.

f_r = Modulus of rupture.

f'_c = 6" X 12" cylinder compressive strength.

C and C_1 are constants ranging from 4.0 to 10.

(c) <u>Modulus of Elasticity</u>

In defining the stress-strain curve of Section 4.2(a), the 1956 ACl Formula,

$$E_c = 1000 f'_c \qquad (4.3)$$

was used to simplify the analysis. This equation gives values of the modulus of elasticity that are low for moderate strength concrete and high for high strength concrete. However, the assumed stress-strain diagram was only used in the computation of the height of flexural cracks for use in a study of Kani's equation for the inclined cracking load (see Section 5.6). The error resulting from Equation 4.3. will be small since the computed crack heights were much more dependent on the assumed stress-strain curve in the tension zone that on the modulus of elasticity.

For other calculations, the more representative value for the modulus of elasticity, E_c, proposed by Hognestad (1951) was used:

$$E_c = 1{,}800{,}000 + 460f'_c \qquad (4.4)$$

Where:

E_c and f'_c are both in pounds per square inch.

4.3 Steel Properties

The reinforcing bars in the tests used for the comparisons were of intermediate or hard grade steel. As the tests procedures are well outlined in ASTM for determining the mechanical properties

it was found that all of the required bar properties were given in the references. The mechanical properties, other than the modulus of elasticity, were therefore taken directly from the reference data. In the case of the modulus of elasticity a mean value of:

$$Es = 29.6 \times 10^6 \text{ psi}$$

was taken for all bars.

CHAPTER V

REVIEW OF INCLINED CRACKING THEORIES

5.1 Introductory Remarks:

This chapter contains a description of the different theories studied in this investigation. It is intended to be used as a reference for the comparisons covered in subsequent chapters and as such, is based entirely on the provisions given by the separate theories as they appeared in the literature. It is proposed to show how the critical shear and diagonal tension stresses are computed according to each of the theories, what significant variables are directly included in the application of each of the theories, and how these variables are taken into account. The limitations imposed on the theories by their authors are also listed.

5.2 Strength of Materials Relationships for Shear Stresses

Reinforced concrete beams are non-homogeneous in that they are made of two entirely different materials. Therefore,

the methods that are used in the analysis of shear and diagonal tension in reinforced concrete beams are different from those used in the design or investigation of beams composed entirely of a homogeneous material such as steel or aluminum. The fundamental principles involved in the derivation of the theories are derived from strength of materials and thus are similar to those relating to homogeneous beams although many modifications are required.

The two fundamental principles which are used either jointly or separately for the development of cracking theories are as follows:

(1) The unit horizontal and transverse shear stresses, v, at any point in an uncracked cross section of a homogeneous elastic beam are given by the equation:

$$v = \frac{VQ}{Ib} \qquad (5.1)$$

Where:

V = Total shear at the section.

Q = Statical moment about the neutral axis of that portion of cross section lying beyond a line through the point in

question and parallel to the neutral axis.

I = Moment of inertia of the cross section about the neutral axis.

b = Width of beam at the given point.

In applying Equation 5.1 to rectangular concrete beams this Equation is often simplified to the approximate form:

$$v = V/bd \qquad (5.2)$$

(2) The principal tension stress, f_{DT}, at a point in a region of combined shear and flexural tension stress is given by the equation:

$$f_{DT} = \frac{f_t}{2} + \sqrt{\frac{(f_t)^2}{4} + v^2} \qquad (5.3)$$

Where:

f_t = Intensity of flexural tension stress.

v = Intensity of vertical or horizontal shearing stress at a point.

Equation (5.3) can be derived from a Mohr's circle.

5.3 Types of Shear Theories

Before discussing the development of each of the separate theories it is possible to group them according to the state of

cracking in the beam being analyzed and the concept of the mechanism of the failure. The oldest school of thought considered horizontal shear as the basic cause of shear failures. In this approach, shear at the neutral axis was taken as a measure of the initial diagonal tension and shear reinforcement was designed as dowels to resist horizontal shearing motions. This theory was essentially abandoned by 1910 and has not been considered in this thesis.

Today, most engineers consider principal tensions to be the basic cause of shear failures. The principal tension cracking theories can be broken down into those considering cracked and uncracked sections. The latter generally recognize that flexural tension stresses exist but also assume that these flexural tension stresses are negligible. In the classical approach as embodied in most building codes since 1920, flexural cracks are assumed to exist but the effect of flexural stresses on the inclined cracking is again assumed to be negligible. In the uncracked and classical cases, then, the magnitude of the shearing stress at the neutral axis can be taken as a measure of the diagonal tension stress.

Considerable work has been done recently in developing principal tension theories for inclined cracking in which the effect of the direct stresses and the shear stresses and, in some cases, the dowel stresses are superimposed to compute the critical principal tension stress. These theories generally deal with a beam which has previously been cracked in flexure. In this thesis we shall refer to this type of theory as a "cracked web" theory to distinguish it from the "uncracked web" case discussed above; where it was assumed that no flexural cracks existed at inclined cracking

Another recent development is the tooth analogy, which states that a reinforced concrete beam under increasing load transforms into a comb-like structure. In the tensile zone the flexural cracks create more or less vertical concrete teeth while the compression zone represents the backbone of the concrete comb. Inclined cracking occurs when the teeth break off due to difference in the tension in the reinforcement of successive sections in the shear span. This theory is developed from observations of long span test members failing in shear.

5.4 Classical Shear Equation (1956 ACl Code Equation)

The 1956 ACl Code states that the shearing unit stress v, as a measure of diagonal tension, in reinforced concrete flexural members shall be computed by Equation 5.4:

$$v = V/bjd \tag{5.4}$$

The development of Equation 5.4 is illustrated in Figure 5.1. If we consider two sections a distance ΔX apart in the shear span, Figure 5.1(a), the assumed state of stress on these sections will be as shown in Figure 5.1(b). From a summation of the moments the difference in steel force ΔT can be expressed as:

$$\Delta T = \Delta M/jd = V.\Delta X/jd \tag{5.5}$$

From a consideration of the horizontal forces acting on a block cut out below the neutral axis as shown in (c), ΔT can be expressed as:

$$\Delta T = vb\Delta X \tag{5.6}$$

Combining these equations gives Equation 5.4. The resulting distribution of shearing stress is as shown in Figure 5.1(d).

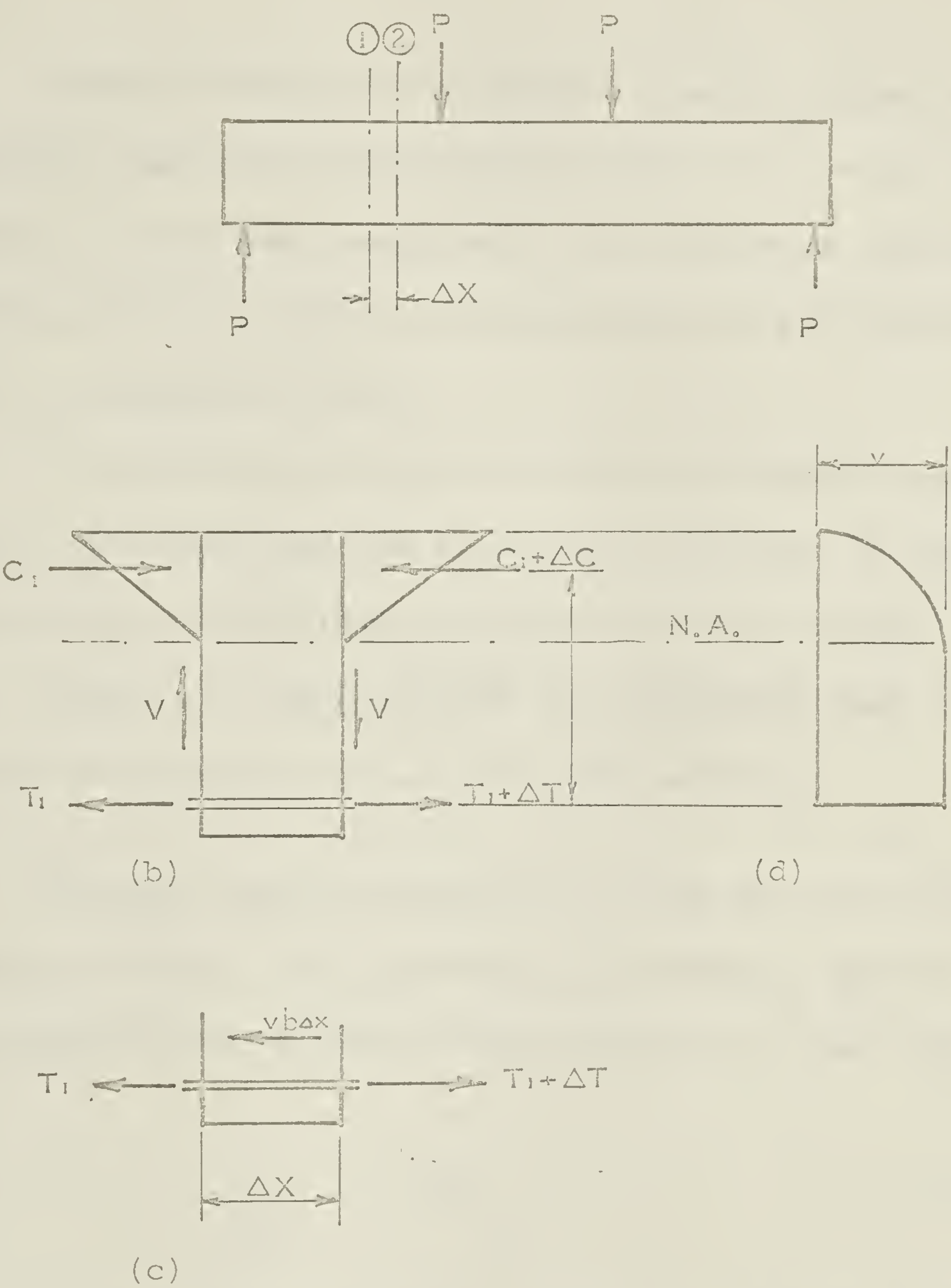


FIGURE 5.1 CLASSICAL PROCEDURE FOR DETERMINING THE HORIZONTAL AND VERTICAL SHEARING STRESS IN REINFORCED CONCRETE FLEXURAL MEMBERS

This is based on the general shearing stress equation assuming, that concrete carries no tensile stresses due to flexure. For these conditions, the horizontal and vertical shearing stresses between the neutral axis and the reinforcement is a constant value.

The problem is not pure shear but rather diagonal tension and the principal tension stress will be equal to the shear stress only if the flexural tension stress is taken equal to zero. This is in keeping with the assumption that flexural tension stresses do not exist in the concrete.

Various codes give various values for the allowable shearing stress. The allowable unit shearing stress v, for beams with no web reinforcement was given in the 1956 ACI Code as:

$$v_{all.} = .03f'_c \tag{5.7}$$

Therefore if the shear capacity of a beam without web reinforcement is assumed to represent the inclined cracking shear, V_{cr}, we can write:

$$V_{cr} = .03f'_c bjd \tag{5.8}$$

5.5 Principal Tension Theories

(a) Uncracked Webs

(i) Guralnick Equation

Guralnick (1959) assumed that the critical stress condition that initiates the first inclined crack is the shearing stress at which failure in pure shear would occur according to Mohr's Rupture Theory. The critical shearing stresses are analyzed at the neutral axis in the gross concrete section because Guralnick assumed that it is admissable to ignore flexural cracking since in most cases very little flexural cracking occurs to damage the beam before the first diagonal tension crack appears. The equation for the cracking shear stress V_{cr}, was according to Guralnick given by:

$$v_{cr} = v'_c = \frac{V_{cr}\ Q}{I\ \ b} \qquad (5.9)$$

Which gives:

$$V_{cr} = \frac{v'_c\ Ib}{Q} \qquad (5.10)$$

Where:

Q = First moment based on the gross concrete section of the cross sectional area lying above or below the centroidal axis.

I	=	The moment of inertia of the gross concrete area.
b	=	The thickness of the beam web.
v'_c	=	The predicted shear stress corresponding to the strength of concrete subjected to pure shear.

Guralnick (1959) assumed that inclined cracking occurred when the stress conditions at the neutral axis of an uncracked beam were equal to those corresponding to failure by the Mohr rupture theory, as shown by circle C'_s in Figure 5.2. To define the Mohr rupture envelope he used the compressive strength, f'_c, and the tensile strength of concrete, f'_t, as shown in Figure 5.2. The Mohr rupture envelope is assumed to be a straight line tangent to the 2 circles, C_c and C_T, rather than the curved line shown in the figure. As a result, Guralnick stated that his method should produce conservative values, and thus defined v'_c using the following equation:

$$v'_c = f'_c \cdot f'_t / f'_c - f'_t \tag{5.11}$$

The tensile strength was evaluated as the modulus of rupture divided by a constant. Guralnick felt that the constant

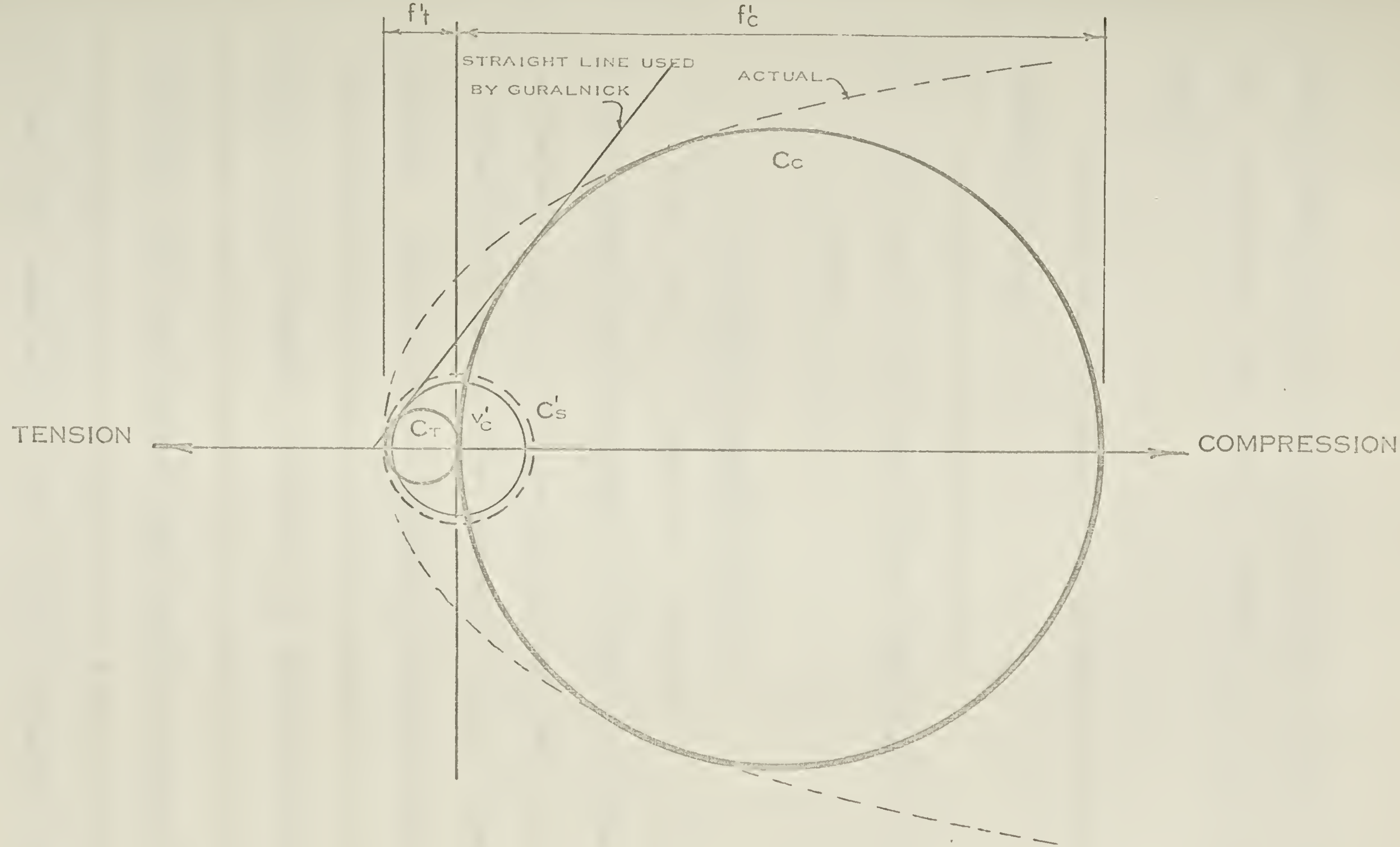


FIGURE 5.2 MOHR'S RUPTURE THEORY ENVELOPE ASSUMED BY GURALNICK IN DERIVATION OF EQUATION 5.11

should range from 1.8 to 2.2 and chose 2.0 as a reasonable value. Thus, he assumed the tensile strength was half the modulus of rupture.

Therefore, the conditions fitting the derivation's assumptions are:

(1) An element at the neutral axis

(2) An element of a deep short beam where f_t is very small in comparison to v.

(ii) Web-Shear Cracking Theory

This theory is based on computations of principal stresses in prestressed concrete beams as presented by MacGregor (1960). A web-shear crack is defined as an inclined crack which develops in the web of a beam before flexural cracks develop in its vicinity. In the case of web-shear cracking, MacGregor stated that the principal tensile stresses computed at the neutral axis of an uncracked I-section will approximate the maximum principal tension stresses in the beam at the time of inclined cracking. This observation was based on computations of the principal tensile stresses along actual inclined cracks at the load immediately prior to the formation of the cracks.

This theory is generally restricted to highly prestressed "I-beams" with short shear spans and thin webs. In computing the web-shear cracking load it is assumed that web-shear cracking is a stress phenomenon and that the critical tensile strength of the concrete in the web is approximately equal to the splitting tensile strength. The principal tensile stresses in the web of a prestressed beam are computed by the usual methods of strength of materials using Equation 5.3 modified to include a term for the prestress assuming the concrete in the beam remains elastic until web-shear cracking.

The effect of the vertical bearing stress acting near the loads and reactions can also be included in this equation if desired. It has been shown by Laupa, Siess and Newmark (1955), that as the shear span is shortened in an attempt to increase the principal tension stresses and decrease the flexural stresses, the shearing forces tend to be transmitted to the supports by the vertical bearing stresses. The zone in which vertical stresses exist extends from 0.7 to 1.0 times the overall depth on either side of the load point and thus if the shear span is shorter than 1.4 to 2.0 times the overall depth,

compressive bearing stresses will affect the principal tensile stresses. Therefore, the principal tension stresses are reduced, as the shear span length is decreased from 2.0 times the overall depth, due to the increased relative importance of the local compressive stresses near the support. The true criterion for determining the location and magnitude of the maximum principal tensile stress in a reinforced beam with a small shear span becomes difficult to determine due to the complex interaction of the shear and flexural stresses and the direct compressive stresses.

As the shear span is increased beyond 1.4H, the flexural stresses have a greater effect and the properties of the beam eventually become such that flexural cracks develop in the shear span. The flexural cracks disrupt the state of stress in the web and as a result, affect the inclined cracking load. As a result the web-shear cracking theory was limited to beams which developed inclined cracks before flexural cracks had developed in the shear-span.

(b) Cracked Webs

(i) Van den Berg Equation

In the derivation of a formula for calculating the

cracking load, Van den Berg (1962) assumed that the formation of the main diagonal crack was due to diagonal tension stresses, $f_{D.T.}$, given by:

$$f_{D.T.} = F_1(V) + F_2(M) \tag{5.12}$$

Where:

$F_1(V)$ = Function of the Shear Force.

$F_2(M)$ = Function of the Bending Moment due to the shear loads.

By incorporating a special loading device Van den Berg (1962) was able to independently vary the bending moment and shear. This made it possible to derive a relationship between the applied shear loads and the corresponding maximum shearing stresses. Plotting the relationship between the shearing force and the measured shearing stresses Van den Berg concluded there was a linear relationship between them. He expressed this relationship with the following:

$$F_1(V) = V/bjd \tag{5.13}$$

Van den Berg further stated that flexural cracking did not affect the distribution of shear across the section to any appreciable degree. Unfortunately, he did not show the relative positions of these flexural cracks and the gages used to measure the shearing stresses.

The conclusions drawn from a statistical analysis of beam tests enabled Van den Berg to suggest a relationship between tension stress or strain in the tension reinforcement and the nominal shearing strength of beams. From pure moment considerations and assuming linear strain distributions, an expression for the strain in the concrete, ε_c, at a point below the neutral axis was given as:

$$\varepsilon_c = \frac{K \cdot M}{jd \cdot A_s \cdot E_s} \tag{5.14}$$

Where:

K = Constant dependent on the position, along the cross section, of the point considered.

Assuming the neutral axis does not change with increasing load, the tension stress in the concrete, f_t, at a point in the middle of the shear span for a simply supported beam was given in the form:

$$f_t = \frac{K.P/2.\ a/2}{jd \cdot A_s \cdot n} \tag{5.15}$$

Where:

f_t = Tensile stress in concrete due to flexure.

$$n = E_s/E_c$$

Van den Berg found that better correlation was obtained by using an artificial internal lever arm, j'd computed directly from the measured steel stresses and moment.

$$j'd = M/\varepsilon_s E_s A_s \qquad (5.16)$$

Where:

ε_s = Measured steel strain

$j'd$ = Empirical internal lever arm depth of tension reinforcement.

$$j' = -0.235 \log (As/b - 0.13) + 0.833 \qquad (5.17)$$

It should be noted that according to this Equation, j' can be larger than 1, presumably because of tension in concrete.

Substituting j'd for jd in Equation 5.15, the following regression equation was calculated for the diagonal tension due to bending moment. The constant 0.116 was determined from experimental results based on the assumption that the main diagonal crack will form when $f_{D.T.}$ is equal to the cylinder splitting strength of the concrete, f'_t.

$$f_{D.T.}(\text{Bending}) = \frac{0.116 . P/2 . a/2}{j'd\ A_s\ n} \qquad (5.18)$$

Thus Equation 5.12 was first developed into the form:

$$f_{D.T.} = \frac{V}{bjd} + \frac{K.P/2.a/2}{jd\ A_s\ n} \qquad (5.19)$$

Then by substituting j = 7/8 in the shear term in Equation 5.13, a formula for the calculation of the cracking load, P_{cr}, was obtained from Equation 5.18 and 5.19, viz.,

$$P_{cr} = \frac{2f'_t + 34}{\dfrac{1}{(7/8)bd} + \dfrac{0.057a}{j'dA_s n}} \qquad (5.20)$$

This equation was intended to apply to beams loaded by concentrated loads.

(ii) <u>ACl-ASCE Committee 326 Equation</u>

ACl-ASCE Committee 326 (1962) assumed that a rational analysis of diagonal tension strength should logically be based on the equation for principal stress at a point, including the effects of flexural tensions and shearing stresses in an equation such as Equation 5.3. It was assumed that the normal tensile stress, f_t, is proportional to the steel stress computed on a basis of cracked-section theory and that the shearing stress, v, is proportional to the average shearing stress where these two quantities may be expressed by:

$$f_t = \frac{C \cdot M}{njpbd^2} = \frac{K_1 M}{pbd^2} \qquad (5.21)$$

And:

$$v = \frac{K_2 V}{bd} \qquad (5.22)$$

The quantities $f_{D.T.}$, n and j at inclined cracking were expressed in terms of the following material properties:

$$f_{D.T.} = 7.5\sqrt{f'_c}$$

$$E_c = 60,000\sqrt{f'_c}$$

$$n = \frac{Es}{Ec} = \frac{Es}{60,000\sqrt{f'_c}}$$

$$j = 7/8$$

$$E_s = 30 \times 10^6 \text{ psi}$$

These values were inserted in Equation 5.3 to give Equation 5.23 which was assumed to express the diagonal tension strength of the section at which diagonal tension cracking begins to form.

$$\frac{V_{CR}}{bd\sqrt{f'_c}} = \frac{1}{\dfrac{C_1 + M \cdot f'_c}{Vpd} + \sqrt{\left(\dfrac{C_1 + M \cdot f'_c}{Vpd}\right)^2 + C_2}} \qquad (5.23)$$

Where:

$$C_1 = \frac{1.53}{10^4} C = \text{Constant}$$

$$C_2 = \left(\frac{F_2}{7.5}\right)^2 = \text{Constant}$$

According to Equation 5.23 the two major parameters governing diagonal tension strength of beams subjected to transverse load only were A and B:

$$A = \frac{V_{Test}}{bd\sqrt{f'_c}}$$

$$B = \frac{M\sqrt{f'c}}{V\ pd}$$

Rather than attempt to evaluate Equation (5.23), Committee 326 derived a design equation based on the simplified form given by Equation 5.24.

$$A = C_1 + C_2 B$$

Or:

$$\frac{Vcr}{bd\sqrt{f'c}} = C_1 + C_2 \frac{Vpd}{M\sqrt{f'c}} \qquad (5.24)$$

Where:

C_1 and C_2 were statistically evaluated from test data as 1.9 and 2500, respectively.

V/M = Ratio of shear to moment at section considered.

Finally, an upper limit of 3.5 was placed on $V/bd\sqrt{f'c}$.

(c) Principal Tension Over Inclined Crack

This analysis, although developed for prestressed members by MacGregor (1960) is included in this thesis as it emphasizes the major affect that an initiating flexural crack has on the principal tensile stresses in its vicinity. Figure 5.3(a) shows a portion of a beam in a region of both shear and moment. In the limiting case, the gap CB-B'C' is made narrower, and thus can be considered to represent the initiating flexural crack.

Considering the forces as those shown in Figure 5.3(b) and assuming no change in the tension reinforcement force, T, crossing the gap, it follows that the change in moment between the sections is accomplished by a shift in the location of the resultant compressive force in the concrete. The stress distribution on the two faces due to flexure is then assumed to be similar to that shown in Figure 5.3(c) and the difference in the stresses on the two faces is represented in Figure 5.3(d).

By assuming that all the shearing forces on the vertical sections act on the uncracked concrete (AB and A'B') and using

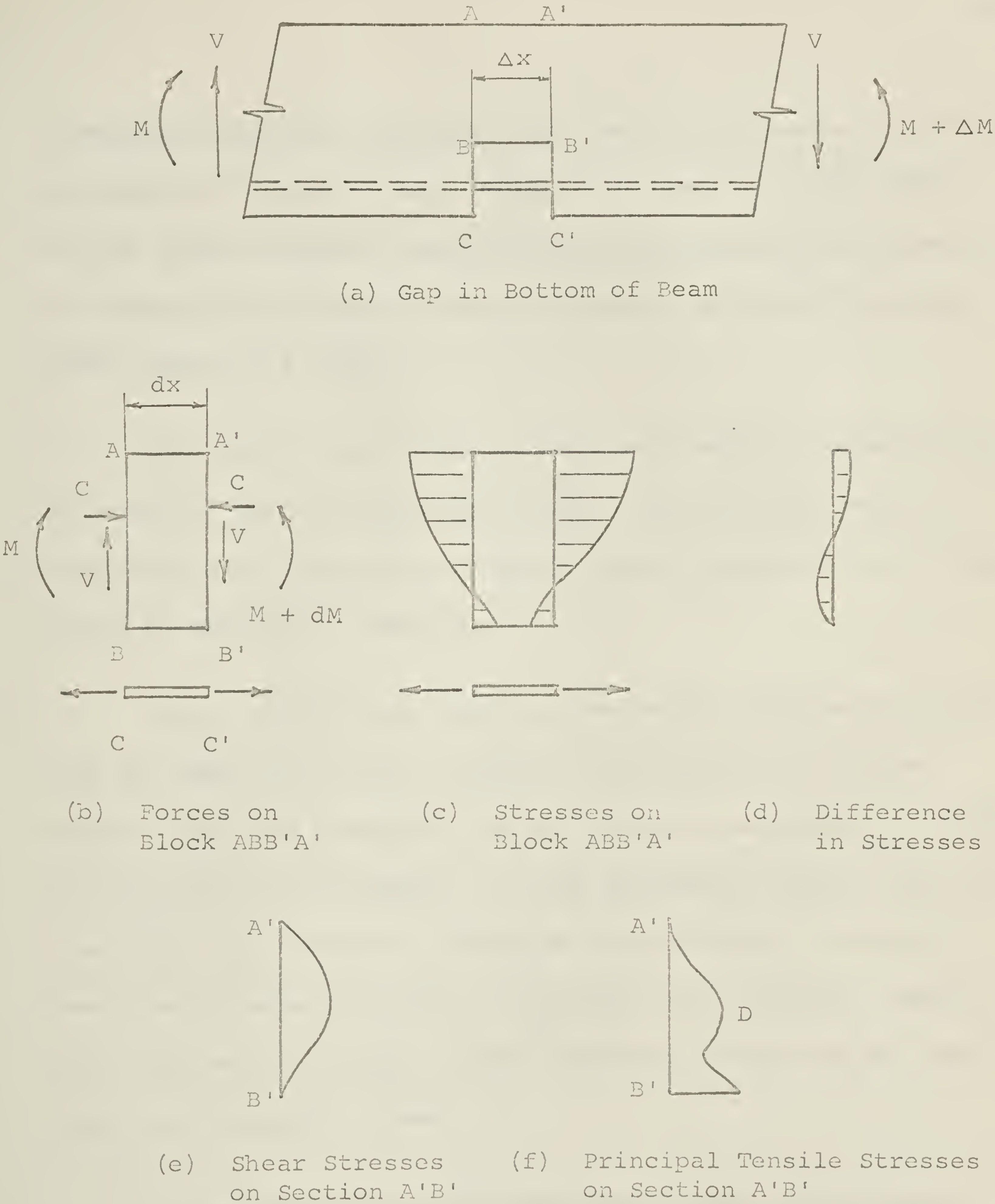


FIGURE 5.3 ANALYSIS OF STRESSES ABOVE A FLEXURAL CRACK

IN A REGION OF COMBINED SHEAR AND FLEXURE

elementary mechanics the resulting shearing stresses are obtained as shown in Figure 5.3(e). Finally, by combining the effect of the direct stresses and the shearing stresses it is possible to compute the principal tensile stresses acting on the face A^1B^1 (Figure 5.3 (f)).

As the flexural crack extends vertically, the principal stresses at Point D increase rapidly and eventually reach a critical value causing an inclined crack resembling a web-shear crack to develop at Point D.

For a given cross section, the height of flexural crack and the magnitude of the stresses corresponding to a given value of applied moment can be evaluated by assuming a concrete stress-strain relationship. If the principal stresses over the crack are computed for a number of crack heights it becomes possible to determine the load at which the principal tensile stresses become critical in the uncracked portion of the beam over the initiating crack.

This theory was never simplified to the point where it could be readily used. In addition, it ignores any shear transfer across the crack by aggregate interlock or doweling, for these reasons it will not be discussed further in this thesis.

5.6 Tooth Failure Theory

(i) Introduction

Similar hypotheses on the mechanism of the formation of inclined cracks in beams with relatively long shear spans have been proposed by Kani (1964) and Moe (1962). Both authors suggested that the critical inclined cracking load was that load at which the concrete element between two flexural cracks in the shear span breaks away from the compression zone of the beam. This element is referred to as a "Tooth".

(ii) Kani Approach

Kani (1964) depicted the internal mechanism of a beam nearing a diagonal tension failure as shown in Figure 5.4. The concrete teeth separated by flexural cracks are loaded by horizontal forces, ΔT, due to the different tension in the reinforcement at the two sections involved. Inclined cracking, is assumed to occur when the horizontal forces break the teeth away from the rest of the beam. This theory is limited to a/d ratios greater than about 2.5 in which range the inclined cracking load actually is the failure load of a beam without stirrups.

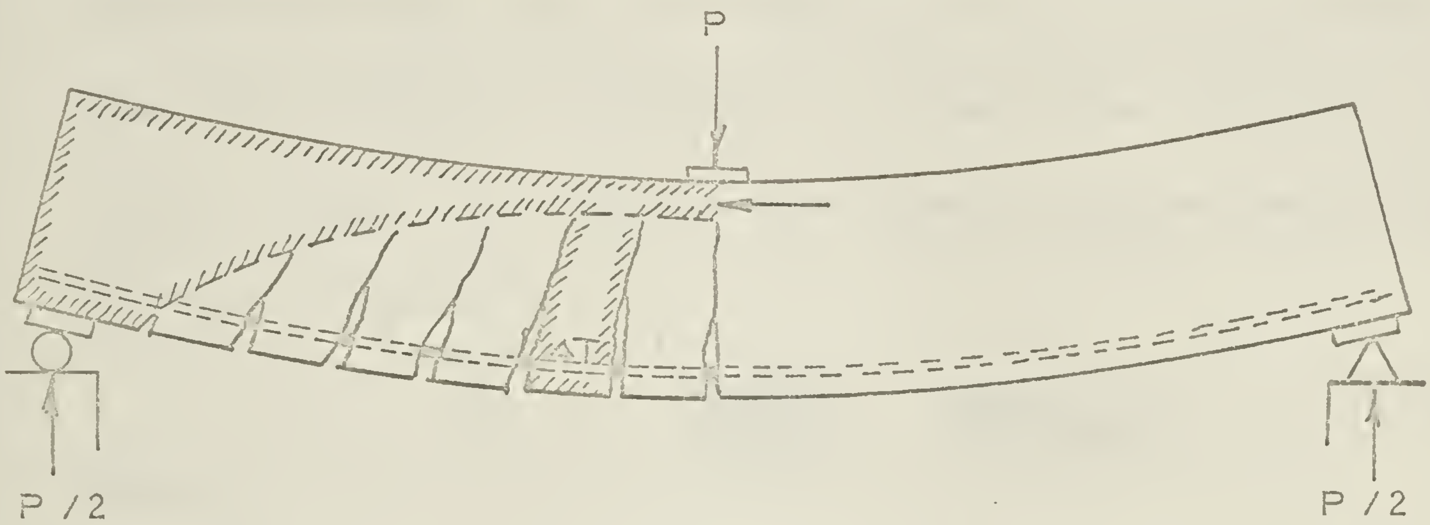


FIGURE 5.4 KANI MECHANISM OF COMB LIKE CONCRETE STRUCTURES

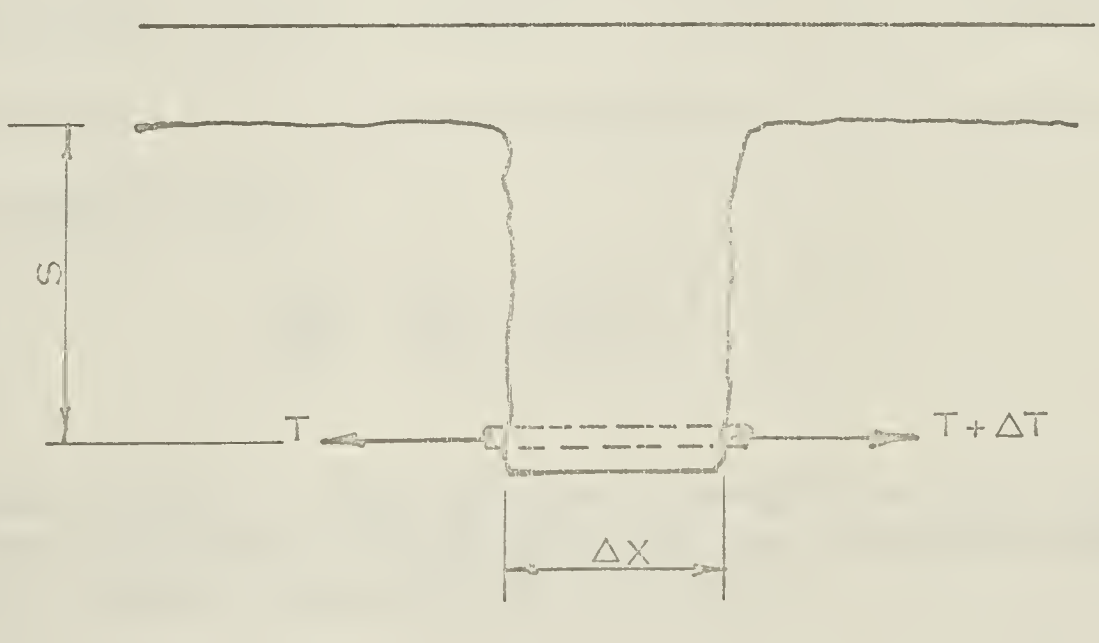


FIGURE 5.5 MOMENT AT FAILURE OF CONCRETE TEETH

The action of a concrete tooth can be compared to that of a short vertical cantilever anchored in the compression zone of the beam and acted upon by the horizontal force, ΔT. When beam theory is applied to a simplified prototype concrete tooth, Figure 5.5, the maximum tensile stress due to the ΔT force is given by:

$$f_t = \frac{M_{TOOTH}}{S_{TOOTH}} = \frac{\Delta T . S}{b . \Delta X^2/6} \qquad (5.25)$$

Where:

S = The mean crack height above the steel.

ΔX = The mean crack spacing.

The maximum resistance to the, ΔT, force occurs when the maximum tensile stress in the tooth, f_t, reaches the modulus of rupture strength, f'_r of the concrete.* Thus the resistance of the concrete teeth per unit length of beam can be expressed as:

$$\frac{\Delta T}{\Delta X} = \frac{f'_t}{6} . \frac{\Delta X}{S} . b \qquad (5.26)$$

* Kani uses f'_t the tensile strength in this relationship, but in private correspondence he indicated that the modulus of rupture was actually intended.

Under increasing load the concrete teeth should break off when this resistance is reached.

Assuming the average horizontal force per unit length, $T/a = \Delta T/\Delta X$ for a beam with constant shear in the shear spans and assuming that the maximum bending moment existing in the central section of the beam is expressed by Equation 5.27, Kani was able to derive Equation 5.28.

$$M_{CR} = Tjd \approx 7/8\ Td \qquad (5.27)$$

$$M_{CR} = \frac{7}{8} \cdot \frac{f'_t}{6} \cdot \frac{\Delta X}{S} \cdot b.a.d \qquad (5.28)$$

Designating that part of the above moment which depends only on the properties of the cross section by M_o, where:

$$M_o = \frac{7}{8} \cdot \frac{f'_t\, bd^2}{6} \qquad (5.29)$$

The critical bending moment where the concrete teeth break away is expressed by:

$$M_{CR} = M_o \cdot \frac{\Delta X}{S} \cdot \frac{a}{d} \qquad (5.30)$$

Kani stated that the critical moment, M_{CR}, given by Equation 5.30 was a linear function of the shear span to depth ratio, a/d. This means that if the cross section, concrete

strength, and the reinforcement are kept constant for a series of beams and only a/d is varied, a linear relationship will be obtained.

The value of M_{CR} increases with increasing a/d, until the full flexural capacity of the cross section is reached. Once this flexural capacity is reached there is no longer any danger of the concrete teeth breaking away and thus no danger of shear failure. The a/d at which the capacity of the concrete teeth reached the flexural capacity is called the "Transition Point".

The important crack factor, $\Delta X/S$, which determines the failure load was to be obtained from tests and has not been defined by Kani (1964).

(iii) Moe Approach

Moe (1962) postulated that the reduction in the inclined cracking load with increasing length of shear span is primarily caused by the detrimental effect of the increasing widths of the bending cracks with increasing ratios of bending moment to shear force, and on this basis he derived the following failure analysis.

Considering equilibrium of the beam element of Figure 5.6 Moe obtains:

$$\Delta T = \frac{V\alpha d}{jd} = \frac{\alpha V}{j} \qquad (5.31)$$

Where ΔT is the unbalanced force from the reinforcement and is the cantilever load on the tooth. The cantilever is also loaded by shear force, V_R, which represents the shear transfer due to the interlocking of aggregate particles across the bending cracks and V_D due to the dowel action of the reinforcement. The total shear is thus carried by two elements: V_R, and V_C where V_C represents the shear carried by the uncracked concrete zone.

Moe's development assumed that the amount of shear transmission across the bending cracks, V_R, decreased gradually as the width, W, of the cracks increased, the total cracked zone shear force which is transmitted by V_R, was given by:

$$V_R = \left(A+(1-A)B/f_s\right)V_t \qquad (5.32)$$

Where:

V_t = the shear force allotted to the cracked zone of the beam according to the classical theory.

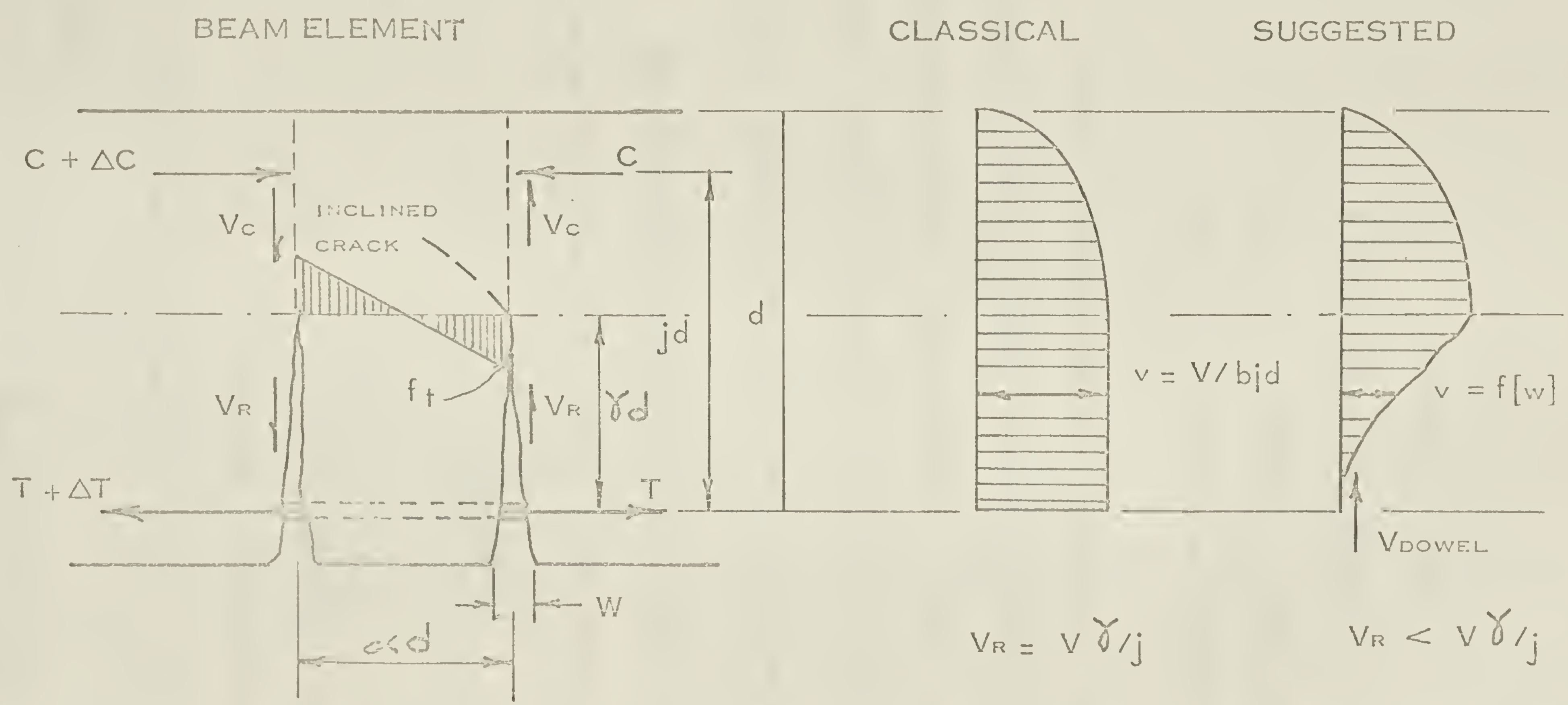


FIGURE 5.6 MOE (1962) MECHANISM OF INCLINED CRACKING FOR LARGE SHEAR SPAN

Or:

$$V_t = vb\gamma d = V\gamma/j \qquad (5.33)$$

A and B are constants

f_s = Stress in the steel reinforcement.

γd = effective height of crack.

In the development of Equation 5.32, it is assumed that the crack width is basically a function of f_s, and secondarily a function of bond, percentage of steel and the quality of concrete. The B constant therefore accounted for these secondary factors and has a stress dimension.

The bending stress in the tooth at the root of the crack can now be computed by simple beam theory.

$$f_t = 6\frac{(T\gamma d - V_R\alpha d)}{b\,\alpha^2\,d^2} \qquad (5.34)$$

Where:

αd = Crack spacing longitudinally.

b = Width of the section.

Substituting Equations 5.31, 5.32, 5.33 into 5.34 Moe obtained:

$$f_t = \frac{f_s - B}{f_s}\ (1-A)\ \frac{6\gamma v}{\alpha j} \qquad (5.35)$$

Where:

$$v = V/bd$$

To simplify this Moe made the following assumptions:

(1) The critical tensile stress causing failure in the cantilever is found at the root of the crack.

(2) This tensile stress is computed with sufficient accuracy by simple beam theory.

(3) The effect of the shearing stresses on the principal stresses causing failure can be neglected.

(4) The tensile strength of the concrete f_t was assumed equal to $k\sqrt{f'c}$

The following expression for the nominal shearing stress at inclined cracking was now found from Equation 5.35.

$$v = \frac{f_s}{f_s - B} \cdot \frac{\alpha j k}{6\gamma(1-A)} \cdot \sqrt{f'c} \qquad (5.36)$$

Moe (1962) did not carry this expression any further, except to note that V_R appeared to be 10 to 20 percent less than the shear force V_t allotted to the cracked zone of the beam according to the classical theory and that the failure

mechanism suggested could not apply to beams with short shear-spans. Because there are four unknown terms, A, B, γ and α, this expression has not been compared or discussed further in this report.

5.7 Flexure-Shear Theory

In reinforced concrete beams, inclined cracks generally start in a region that previously has been cracked in flexure. Studies of prestressed concrete beams by MacGregor (1960) and Sozen and Hawkins (1963) indicated that the flexural cracks strongly influenced the development of subsequent inclined cracks. This type of inclined crack was called "Flexure-Shear Cracking".

Figure 5.7 taken from Sozen and Hawkins (1963) showed the data on flexure-shear cracking for 146 simply supported prestressed concrete beams tested at the University of Illinois. The shear corresponding to the formation of the flexure-shear crack was found to be quite sensitive to the shear, V_f, required to cause a flexural crack at a given point in the shear span. Thus, the flexure-shear cracking load was expressed as:

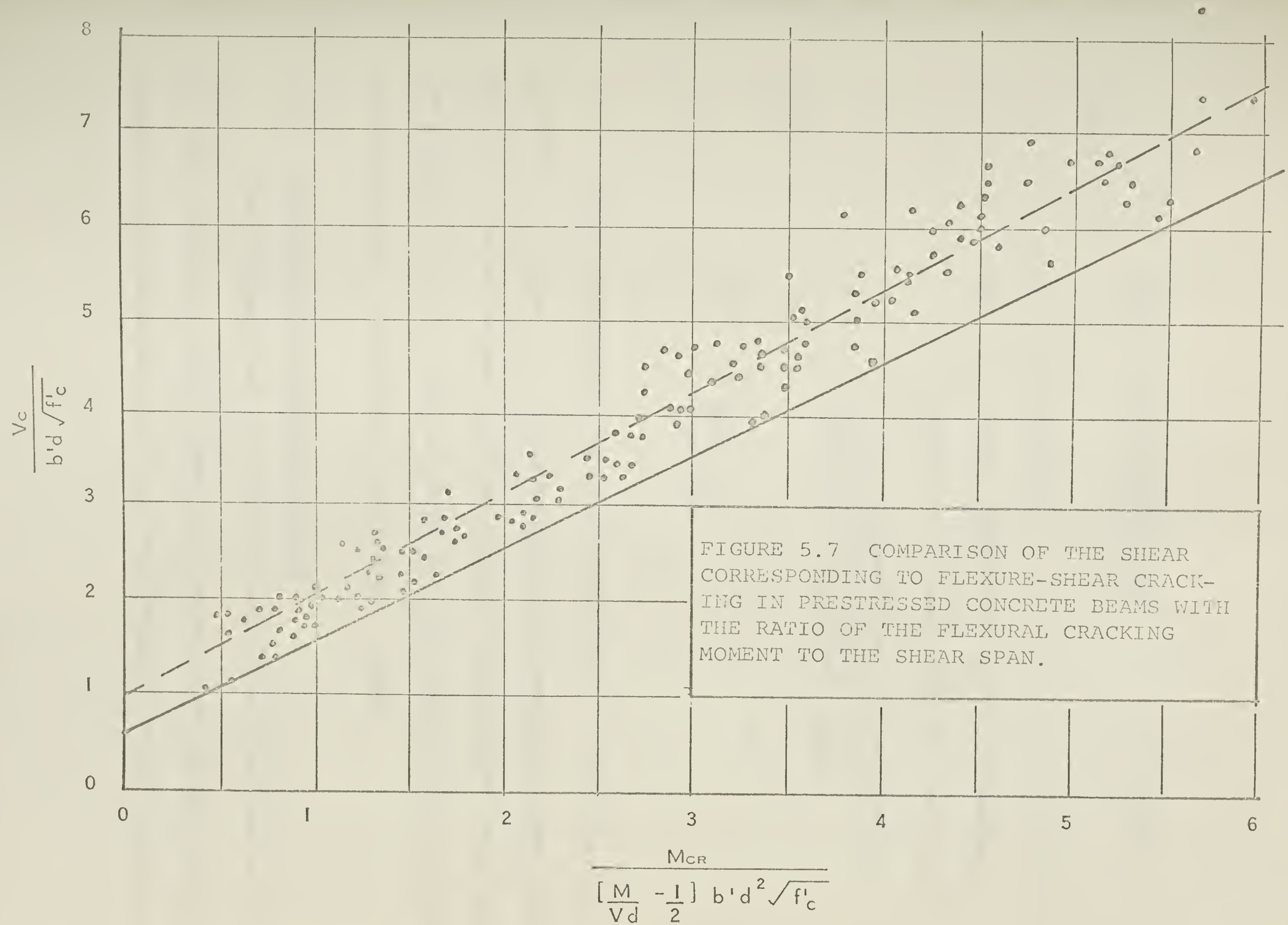


FIGURE 5.7 COMPARISON OF THE SHEAR CORRESPONDING TO FLEXURE-SHEAR CRACKING IN PRESTRESSED CONCRETE BEAMS WITH THE RATIO OF THE FLEXURAL CRACKING MOMENT TO THE SHEAR SPAN.

$$V_c = C.b'd\sqrt{f'c} + V_f \qquad (5.37)$$

Where:

C = Constant

V_f = Shear required to cause a flexural crack at d/2 away from the load point (this flexural crack is assumed to initiate the inclined crack).

Or:

$$V_f = \frac{M_{cr}}{\frac{M}{V} - 0.5d}$$

Sozen and Hawkins rearranged Equation 5.37, to the form plotted in Figure 5.7. This equation represented a lower bound to their flexure-shear cracking data for prestressed beams and is specified in the 1963 ACl Code for the design of prestressed-concrete beams developing flexure-shear cracks.

5.8 Miscellaneous Shear Equations

For completeness, a number of empirically derived equations for the inclined cracking load will be outlined in this section. However these equations will not be discussed to any great extent.

(a) Clark

Clark (1951) found that the position of loads on a beam influenced considerably the shear carrying capacity of the beam. He introduced the term a/d as a means of expressing this span-to-depth ratio.

The following empirical expression was developed by Clark:

$$v_c = 7000p + (0.12f'c)\frac{d}{a} + 2500\sqrt{r} \quad (5.38)$$

Where:

v_c = Calculated shearing stress at Maximum load.

p = $\frac{As}{bd}$ = Steel ratio of longitudinal tension reinforcement.

r = $\frac{A_v}{bs}$ = Ratio web reinforcement.

s = Spacing of stirrups.

This empirically derived formula and its inclusion of the a/d ratio was a major step.

(b) Sozen, Zwoyer and Siess
(University of Illinois)

The analysis and equations developed by Sozen, Zwoyer and Siess (1959) were based on experiments on prestressed beams but it was suggested that similar expressions could be used for normal reinforced concrete beams. The data on inclined cracking loads were expressed as functions of the various factors expected to affect the stresses in the beams and compared with various parameters involving variables expected to contribute to the strength of the beam. The final grouping of the parameters grew out of a principal tension analysis. The following relationship was found for prestressed beams:

$$\frac{Mcr}{f_t bd^2\sqrt{b'/b}} = \frac{Fse}{A_g f_t} + 1 \qquad (5.39)$$

Where:

- Mcr = Moment at inclined cracking defined as the product of the applied shear at the inclined tension cracking load and the length of the shear span.
- f_t = Assumed tensile strength of concrete.
- b = Top flange width.
- b' = Web thickness.

d = Effective depth.

Fse = Effective prestress force.

Ag = Gross area of cross section.

For reinforced concrete beams, the left-hand term of Equation 5.39 would be significant because the prestress term, $\frac{Fse}{Agf_t}$, drops out. Expansion of the left term gives the following:

$$\frac{Mcr}{f_t bd^2\sqrt{b'/d}} = \left(\frac{Vcr}{f_t bd}\right)\left(\frac{1}{\sqrt{b'/b}}\right)\left(\frac{a}{d}\right) \qquad (5.40)$$

Where:

Vcr = Applied shear at inclined tension cracking.

a = length of shear span.

Therefore, the first term in parenthese, $V_{cr}/f_t bd$, measured the effect of shear. The, b'/b, term reflected the effect of the web thickness on the cracking load and the last term, a/d, represented the effect of bending moment. This equation differs from the more recent inclined cracking equations in that it considered the effect of the web width and neglected any effect of steel percentage, although the prestress term may have indirectly included any effect of the steel percentage.

(c) Whitney

Whitney (1957) observed from tests on stub beams and knee frames that the shear strength appeared to depend on the ultimate flexural capacity of the member and on the ratio of shear span to depth. He assumed that shear was largely a function of the strength of the reinforcement and the proportions of the beam and that the unit shear, v, should be considered as V/bd instead of V/bjd. In addition it was assumed that the ultimate shear strength, v_u, for beams without web reinforcement should be based on diagonal cracking load and not on the maximum load carried by beam.

These observations led to the development of Whitney's expression for ultimate shear, v_u, in a simple beam as:

$$v_u = 50 + 0.3\frac{Mu}{d^2}\sqrt{d/ls} \qquad (5.41)$$

Where:

Mu = The ultimate moment capacity per inch width of beam.

ls = The shear span.

d = Depth to the steel.

Whitney (1958) withdrew the above equation on the basis of further test data and a re-evaluation of the major parameters. The term Mu indirectly measured the steel percentage since the majority of the beams checked with this equation had reinforcement yeild points in the order of 40 to 50 ksi.

CHAPTER VI

EXAMINATION OF INCLINED CRACKING EXPRESSIONS

6.1 Introductory Remarks:

Having outlined the different inclined cracking expressions and their basic assumptions, it is now possible to discuss the relative merits of these theories and to compare them with observed behavior. The choice of the significant variables is studied and the manner in which each of the expressions fits the tests used by ACl-ASCE Committee 326(1962) in deriving their expression is discussed.

In the numerical evaluation of the expressions, tests on four types of non-web reinforced rectangular beams are considered:

(1) Simple beams with one and two concentrated loads.

(2) Stub beams with one concentrated load.

(3) Restrained beams loaded symmetrically at the overhangs and with one or two concentrated loads in the center span.

(4) Two span continuous beams under different arrangements of concentrated loads.

The details of the tests are discussed or referenced in the ACl-ASCE Committee 326 Report.

Each hypothesis was assumed to apply to all loading conditions, support conditions and cross sections in the test data unless otherwise limited by the author of the theory. Thus, for example, Kani's tooth failure hypothesis was limited to relatively long shear span lengths.

6.2 Classical Shear Equation
(1956 ACl Code Equation; $Vc = .03f'_c bjd$)

For nearly forty years building codes have expressed the inclined cracking load using an equation of the form:

$$V_{cr} = Cf'_c bjd \qquad (6.1)$$

In this equation the nominal shear V, is taken as a measure of the diagonal tension in the beam.

Equation 6.1 and the theory it is based on fail to accurately predict either the behavior or the strength of beams developing inclined cracks. The assumption that the shear stress is equal to the diagonal tension stress, although valid

at the neutral axis of an uncracked elastic beam, ignores the flexural tension stresses below the neutral axis and any stress concentrations which might develop in the vicinity of flexural cracks.

The diagonal tension or shear strength of a beam is not a simple linear function of the compressive strength of the concrete. As pointed out in Chapter 3, diagonal tension strength depends very much on the shear ratio, a/d, and to a lesser degree on the steel ratio; neither of these variables is represented in Equation 6.1.

In comparing the Classical Equation with test results, the dimensionless measure of diagonal tension strength represented by $V/bd\sqrt{f'c}$ was obtained by a simple transformation and by assuming j equal to 7/8. The poor representation of test data by the equation is indicatd by the high value of the mean ratio of measured to calculated shears and the very high coefficient of variation given in Table 6.1. It was due to the inability of Equation 6.1 to provide a measure of diagonal tension strength that a joint committee of ASCE and ACl was formed with the assignment of developing methods for designing reinforced concrete

members to resist shear and diagonal tension consistent with ultimate strength methods.

TABLE 6.1

MEANS AND COEFFICIENTS OF VARIATION OF TEST BEAMS WITHOUT WEB REINFORCEMENT ACCORDING TO THE DIFFERENT THEORIES

Type and Designation of Theory	Mean Value of $\frac{V_{Measured}}{V_{Calculated}}$	Coefficient of Variation Percent
(1) Classical Shear Equation (1956 ACl Code Equation)	2.000	26.8%
(2) Uncracked Webs		
(i) Guralnick Equation	1.053	21.9%
(3) Cracked Webs		
(i) Van den Berg Equation	.983	19.7%
(ii) ACl-ASCE Committee 326 1963 ACl Code Equation	1.097	15.1%
(4) Tooth Failure		
(i) Kani Equation*	1.165	44.8%
(5) Flexure-Shear		
(1) Using Gross Section	.998	17.5%
(2) Using Transformed Section	1.000	16.0%
(3) Transformed Section Plus Doweling	1.017	15.5%

* A limited number of test beams correlated.

6.3 Principal Tension Theories

(a) Principal tensions in uncracked webs.

(i) Guralnick Equation:

Guralnick (1959) assumed that inclined cracking would occur when the stress conditions at the neutral axis of an uncracked beam were equal to those corresponding to failure by the Mohr Rupture envelope for the "pure-shear" case.

Although Guralnick's equation is easy to solve for any beam it does not reflect the manner in which inclined cracks seem to develop. The conditions for first diagonal cracking are assumed to exist in an element at the neutral axis. The concrete below the neutral axis is assumed to have no tensile stress due to flexure, yet the concrete tensile strength is used to determine the allowable shear stress v'_c at the neutral axis as computed by Equation 5.11.

Laboratory tests and observations of regular reinforced concrete beams indicate that first diagonal cracking forms at locations where f_t is a sizeable quantity in comparison to v and that in beams with a medium to long shear span to depth ratio, flexural cracking will have occurred by the time the

inclined cracks develop.

Guralnick does consider the strength of concrete in tension and does consider it a separate variable in determining the shear v'_c. This determination of v'_c seems to be the major difference between his approach and the classical derivation for rectangular beams.

In the numerical comparison of Guralnick's Equation 5.8 with the ACl 326 test results in this thesis, the value of f'_t was assumed to be dependent on the value of the square root of the concrete compressive strength since tensile strength was not reported in most of the test data. A value of $f'_t = 4.0\sqrt{f'_c}$ was found to give the best mean ratio of measured to calculated values of $V/bd\sqrt{f'c}$. This mean ratio and the corresponding coefficient of variation are given in Table 6.1. The large coefficient of variation suggests that this method does not account for all the factors involved.

(ii) <u>Web-Shear Cracking Theory</u>

As was pointed out earlier, web-shear cracks are practically limited to prestressed concrete beams and will

seldom if every occur in reinforced concrete beams. Consider, for example, a simple rectangular reinforced concrete beam with a shear span 1.5 times the overall depth. This beam would have as high a ratio of web-shear to flexural cracking load as is practically possible in a rectangular beam. Even in this extreme case, however, the computed web-shear cracking load is 1.8 times the load required to cause an initiating flexure crack at the middle of the shear span. In determining the web-shear cracking load for this example the vertical compressive stresses were neglected and as a result the load is conservative. For I-Beams with thin webs the ratio of web-shear cracking load to the load causing the initiating flexural crack will decrease. Thus, for the same span and loading, the computed web-shear cracking load will drop to a value approximately equal to the flexural cracking load for a symmetrical I-Beam with a web thickness of 30 percent of the flange width.

The web-shear cracking theory has not been compared to the test data because it obviously does not apply to reinforced concrete beams of the type tested.

(b) Principal Tensions in Cracked Webs

(i) General Both the ACl 326 Equation and the equation developed by Van den Berg (1962) are very similar in development, in that a diagonal tension strength analysis is used, based on principal stresses at a point. In both methods the variables assumed to affect this strength were grouped into two separate parameters with statistically derived coefficients. Flexural cracking is taken into account in a semi-rational manner in the expression for the inclined cracking load. Both methods of analysis were based on an assumed flexural crack location, with no attempt made to develop a method of determining the critical crack location. The assumed crack location is not unreasonable however.

(ii) Van den Berg Equation

Van den Berg (1962) assumed that the formation of the main diagonal crack was due to principal tension stresses above or adjacent to the crack. Based on data obtained from his investigation Van den Berg concluded that the principal tension stress was a function of the shear stress and a function of the flexural stress in the concrete as measured by the

strain in the tension reinforcement due to moment. The derivaation of this expression for the calculation of the diagonal cracking load was presented in Chapter 5.

The utilization of restrained beams with applied end moments appears to be a new approach for determining the effect of shear loads on diagonal tension stress. Further tests for a range of variables under similar load conditions would be desirable to substantiate Van den Berg's conclusion that there was a linear relationship between the applied pure shear force and the shearing stress, as given by Equation 5.13.

The superposition of the diagonal tensile stress due to some function of bending moment and the diagonal tensile stress due to some function of pure shear to obtain the maximum diagonal tension was based on the observation that flexural cracking did not affect the shear stress distribution prior to the formation of the main inclined crack. This observation differs from the consensus of MacGregor (1960), Moe (1962), Sozen and Hawkins (1963) and others. These investigators indicated that cracks which form in the shear span prior to the formation of the main diagonal crack act as stress raisers, causing

high stress concentrations at the ends of these cracks, which aid in triggering the formation of the main diagonal crack or determine its position to a certain extent. Further investigation is, however, necessary to provide proof for the above conditions as neither side is substantiated by actual test data.

In applying the cracking Equation 5.20 developed by Van den Berg, to the ACl tests, it was found that the empirical expression for the internal lever arm coefficient j' restricted the cracking load equation to beams with As/b ratios greater than 0.2. In addition, Van den Berg's expression was not dimensionally correct. To allow a wider comparison with test data a more general expression for j' was derived based on percent tension reinforcement. Using the same test beam values as Van den Berg, the relationship between j' and steel percentage plotted in Figure 6.1 was derived. It can be expressed as

$$j' = -.062p + 1.155 \qquad (6.2)$$

Where:

$$j' = 1.1 \qquad p < .85\%$$

$$j' = .875 \qquad p > 4.50\%$$

The limits of Equation 6.2 are based on Equation 3.16 and that concrete would resist comparatively little tensile

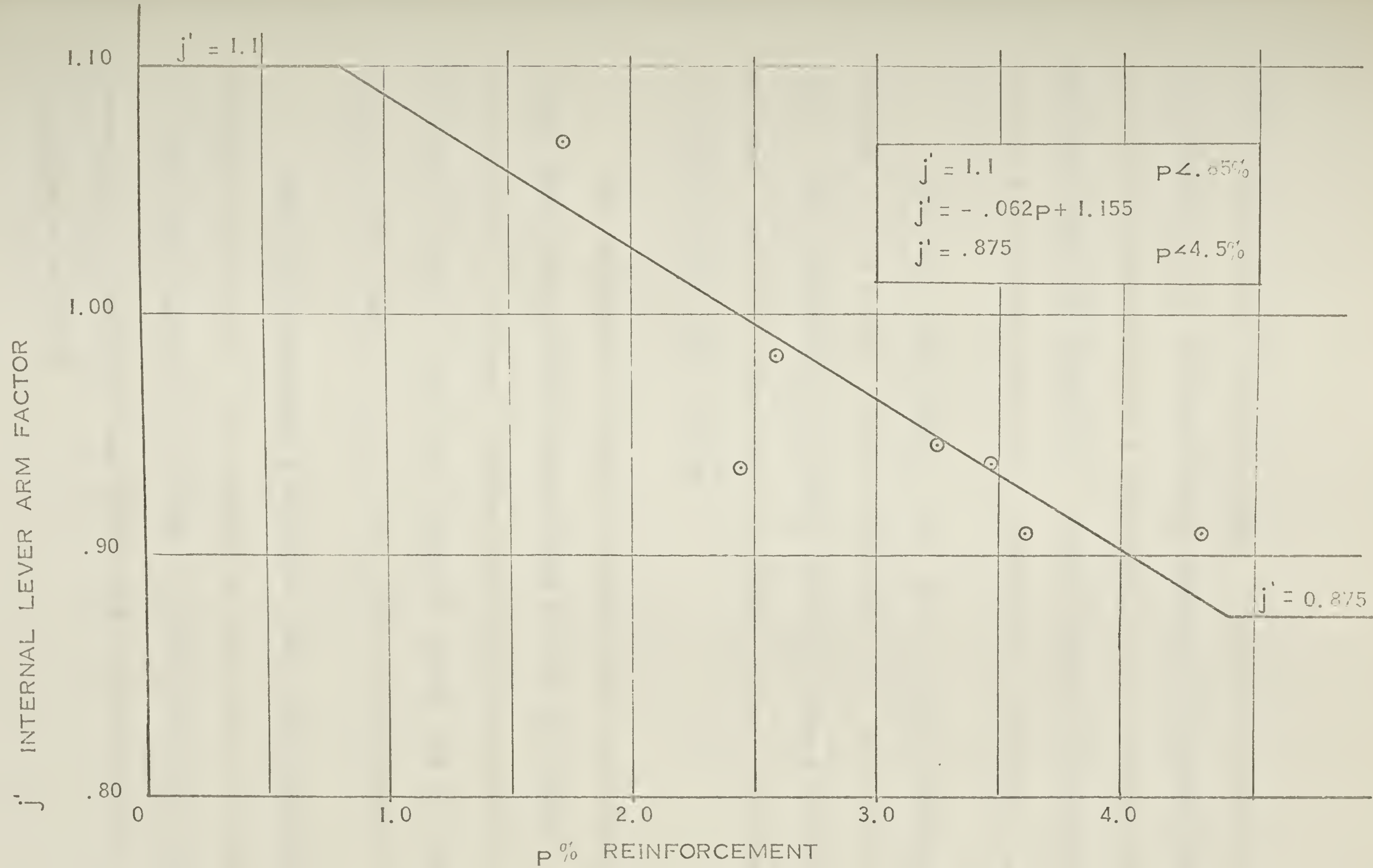


FIGURE 6.1 THE RELATIONSHIP BETWEEN INTERNAL LEVER ARM AND PERCENT REINFORCEMENT

stress for over reinforced beams. For small amounts of reinforcement, j' has values greater than 1.00 because the contribution of the tension force developed in the concrete below the neutral axis was neglected. These concrete tension forces become significant when the reinforcing percentage is low.

Van den Berg assumes that the critical diagonal crack occurs at the middle of the shear-span for concentrated loading. This assumption is fairly good for beams having medium to small ratio of shear-span to depth but may be unconservative in longer shear span ratios.

The shearing strength of beams under uniformly distributed loads were also briefly considered by Van den Berg. It was stated that a similar cracking load equation could be obtained if the position of the main diagonal crack and the corresponding critical combination of shear and bending moment were known.

When Van den Berg's Equation (Eq 5.20), modified by using Equation 6.2 for j', is compared to the test data published in the ACl-ASCE Committee 326 Report (1962), the mean value of measured to calculated was 0.983 and the coefficient of variation was 19.7%.

(iii) <u>ACl 326 Equation</u>

The basic criteria used by Committee 326 for the analysis of beams without web reinforcement was that the diagonal tension strength was a function of Equation 5.3 for the principal stress at a point. Due to the difficulties in the direct determination of the tensile bending stress, f_t, and the shearing stress v, in this equation, Committee 326 evaluated these stresses statistically on the basis of the critical diagonal cracking load as observed in tests. After a wide study of test results the committee concluded that the primary variables affecting these stresses were:

(1) Ratio of shear to moment, M/Vd.

(2) Concrete compressive strength, f'_c.

(3) Percentage of longitudinal reinforcement.

The derivation of the ACl-ASCE Committe 326 equation was traced previously in Section 5.5(b)-ii. In its final equation, (Eq. 5.24), Committee 326 presented a semi-rational expression for the shear at inclined cracking.

In the above mentioned expression the magnitude of the tensile bending stress (basis of Parameter B) is computed on the assumption of cracked section theory; the unit shear stress

(basis of Parameter A) is expressed as the shear stress according to the classical theory which is in effect the average shear between two cracks rather than the shear adjacent to the flexural crack. The moment to shear ratio to be used in the final equation should be taken at the section of diagonal tension cracking which is not known. As a result it was necessary to assume the crack location.

In diagonal tension or shear failures, the first stage of failure is in effect a failure of the central or web portion of the beam in tension. Chapter IV discussed and pointed to the variation of results and errors that can occur when concrete compressive strengths are used to determine concrete tensile properties. Committee 326 expressed concrete tensile strength in terms of the square root of compressive strength of concrete which more closely represents the trend of the tensile strength than the compressive strength does.

The above discussed assumptions and relationships used by Committee 326 therefore indicate that the design equation presented is open for possible improvement when the behavior of beams failing in shear is more clearly understood.

Table 6.2 gives the mean values of tested to calculated and the coefficients of variation as they appeared in the report of Committee 326. Table 6.1 gives the values for similar means and coefficients considering only the 194 test beams used in the derivation of the ACI-ASCE equation.

TABLE 6.2

COMPARISON OF RESULTS OF 430
TESTS WITHOUT WEB REINFORCEMENT
ACCORDING TO EQUATION 5.24

Type of test beam	Cross Section	Loading	No. of test beams	Test/Calc	Coefficient of variation, percent
Beams used in derivation of Eq. (5.22)					
Simple	Rectangular	Concentrated	45	1.076	15.8
Simple with stub	Rectangular	Concentrated	48	1.239	13.5
Restrained	Rectangular	Concentrated	86	1.041	8.4
Continuous	Rectangular	Concentrated	15	1.031	13.0
Other beams					
Simple	Rectangular	Uniform	64	1.192	10.9
Simple	Rectangular	Concentrated	124	1.300	14.9
Restrained and continuous	Rectangular	Concentrated	14	1.091	13.8
Simple and restrained	T-beams	Concentrated and uniform	34	1.221	19.6
All beams			430	1.180	16.2

6.4 Kani Method

In a paper published in 1964 Dr. Kani proposed an analogy between inclined cracking in reinforced concrete beams and the failure of the teeth of a comb when subjected to a lateral load. In the case of the reinforced concrete beam the tooth is the concrete element between two flexural cracks and the load on the tooth is the difference in steel stress between the two sections, Kani separates the failure conditions for short shear spans, where bearing stresses cannot be ignored (a/d ratios less than 2.5) and longer shear spans by incorporating a subdivision and applying a tied arch approach to the short shear span ratios. He does not attempt to define the inclined cracking load for beams developing a tied arch failure mechanism.

Perhaps the most basic criticism of the tooth failure mechanism is that it does not correctly portray the shape of the inclined crack that forms. Kani in his development accents the crack height, S, in that "the breaking away of a concrete tooth" is determined by a sharp turn of the crack into the direction of the load P. For shear spans having an a/d ratio larger than about 4, the crack pattern does somewhat resemble

teeth broken off at the top Figure 6.2, but this is certainly not the case for a/d ratios of 3.5 or less, Figure 6.3 . Referring to the latter figure and to most crack patterns shown in the literature it is doubtful whether the concrete teeth ever break off, thus the crack pattern assumed by Kani is purely hypothetical. Actual cracking in the shear span, as was discussed in Chapter II and in parts of Chapter III, is inclined and is the result of flexural stresses, shear stresses and probably some tooth stresses acting on the same element.

Kani's equation does show agreement with conclusions made by Van den Berg (1962); Van den Berg's tests suggested that the inclined cracking load was more strongly a function of the rate of change of moment (or shear) in the shear span than a function of the magnitude of the moment. This, then agrees with Kani's equation since the ΔT forces are a function of the change in moment rather than the maximum moment.

The basis for numerical calculations in the tooth failure theory proposed by Kani requires that the spacing ΔX and the height of crack, S, in Equation 5.30 be known. In his development Dr. Kani does not define a method by which ΔX and S might

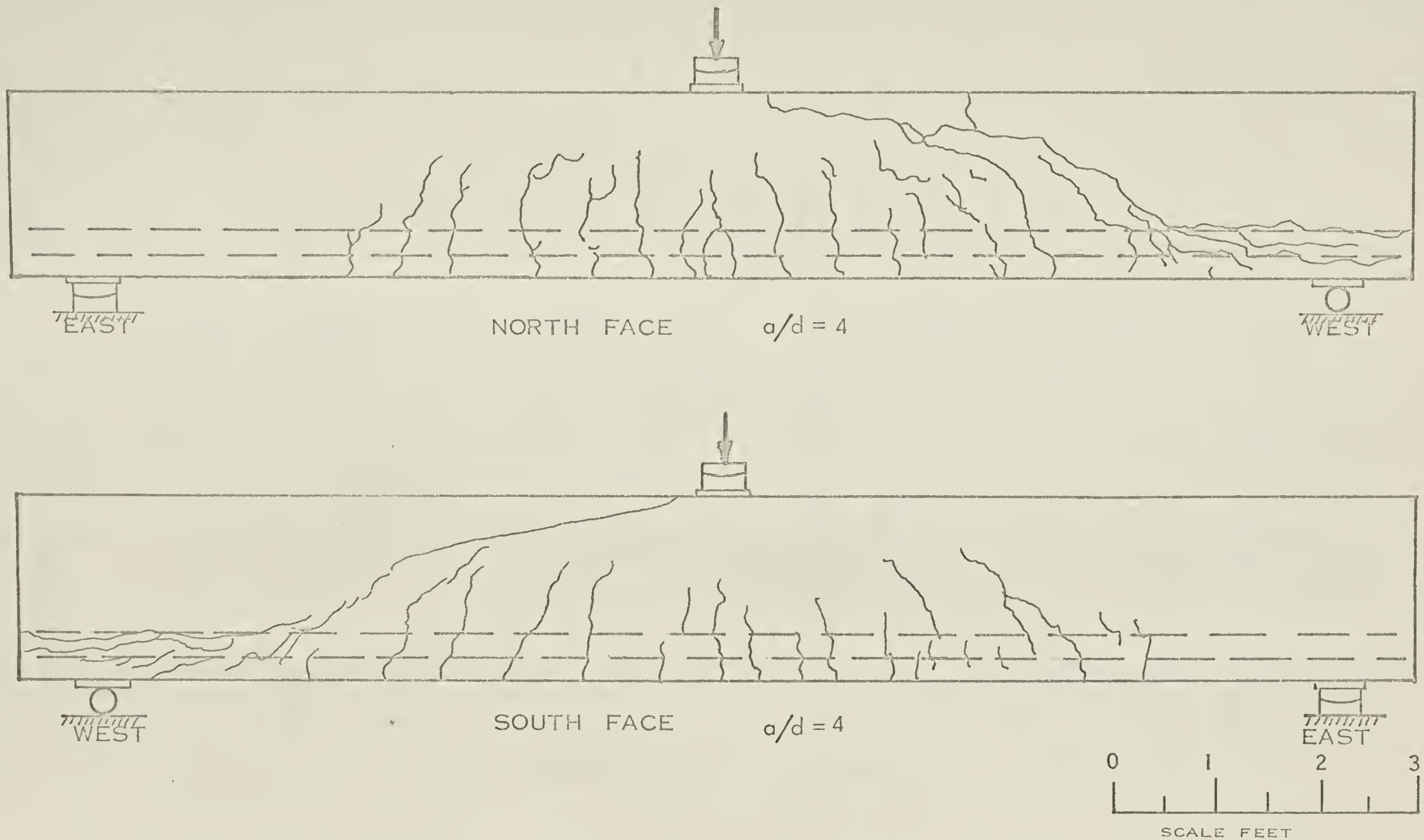
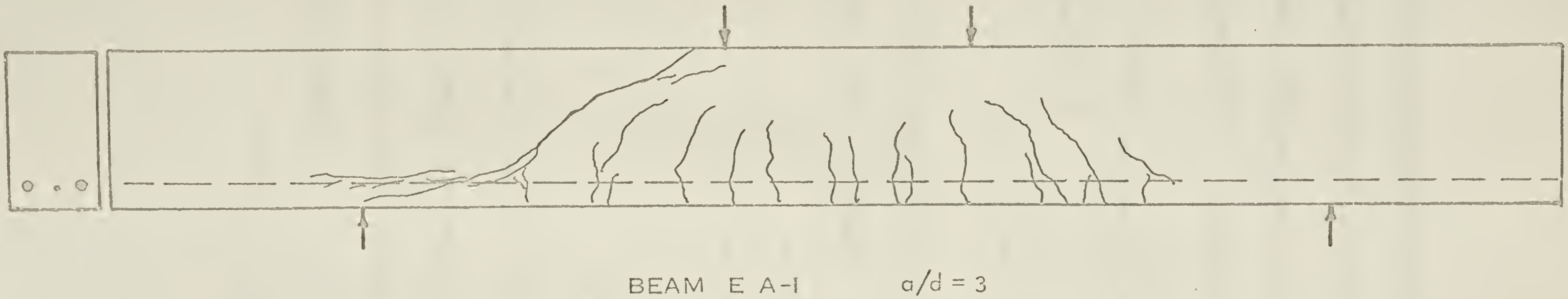


FIGURE 6.2 BEAM OB-1 - FROM BRESLER AND SCORDELIS (1964)



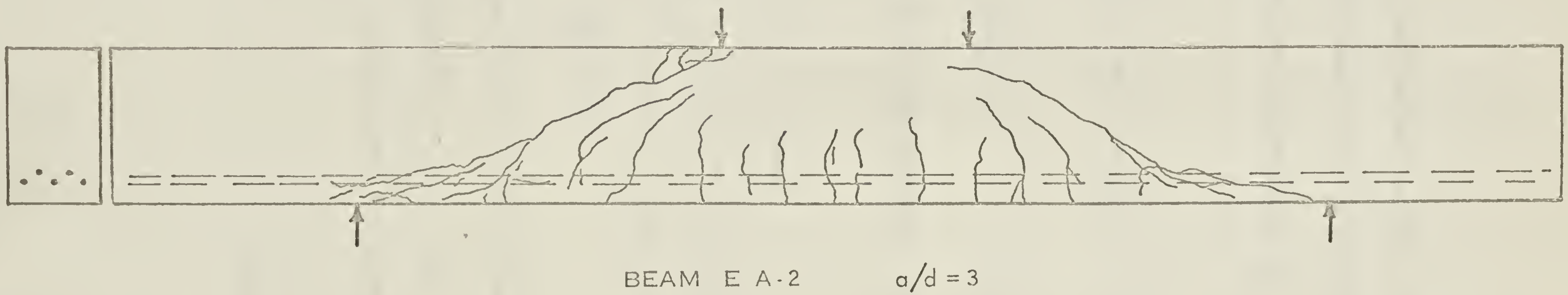


FIGURE 6.3 BEAMS EA-1 AND EA-2 FROM LEONHARDT AND WALTHER TESTS

- (NATIONAL RESEARCH COUNCIL OF CANADA - TECHNICAL TRANSLATION 1172)

be estimated other than by actual test measurement. Evaluation of Equation 5.30 for comparison purposes with ACl-ASCE Committee 326 tests, therefore, requires that an expression for the crack factor, $\Delta X/S$, be found or derived.

The following expression for crack height, S, is derived in Appendix A:

$$S = \left(-.08 + 0.1\frac{26f'c}{P}\right)d \qquad (6.2)$$

This expression is limited to f'c/p greater than 2.0 and less than 4.0 where

d = effective depth of beam.

f'c = compressive strength of concrete in kip/in^2.

p = tension reinforcement ratio in percent

that is $p = \frac{As}{bd} \times 100$

The expression for crack spacing, ΔX, developed by M. Bruce (1960) is rearranged in Appendix A such that the average crack spacing can be given as follows for simple rectangular beams:

$$\Delta X_{Ave.} = \frac{1.88b(h-d)}{\Sigma o} \qquad (6.3)$$

The term $\triangle X/S$, as determined from the above equations incorporates four variables, affecting diagonal tension strength, namely percentage of longitudinal reinforcement, concrete compressive strength, bond strength and beam size. The Equations 6.2 and 6.3 are, however, limited in scope to lightly reinforced beams with concentrated loads. The effects of axial load have not been considered, although $\triangle X/S$, will be affected.

In deriving the relationship for height of cracks, S, only simply supported beams with tension reinforcement only were considered. The validity of the expression was also limited to f'c/p ratios greater than 2.0 and less than 4.0 thus limiting the final comparison with the ACl-ASCE test results to only 20 test beams.*

* Test beams satisfied all the following limits

(1) Simply supported with concentrated load.
(2) Shear-span to depth ratios greater than 3.0
(3) The ratio of concrete compressive strength (ksi) to steel ratio (%) greater than 2.0 and less than 4.0.

By use of a parabolic equation for "S" values of f'c/D greater than 4 could be considered.

In comparing Equation 5.30 with these limited test results, the mean value of $\Delta X/S$ had to be taken according to Kani (1964) therefore the crack height as expressed by Equation 6.2 was replaced by S/2, so that Equation 5.30 appeared as,

$$\frac{V_{cr}}{bd\sqrt{f'c}} = \frac{7}{8} \cdot \frac{7.5}{6} \cdot \frac{2\Delta X}{S} = 2.19\frac{\Delta X}{S} \qquad (6.4)$$

When:

$$f'_t = 7.5\sqrt{f'c}$$

and where ΔX and S are computed according to Equations 6.2 and 6.3.

The results obtained using Equation 6.4 are indicated in Table 6.1. The coefficient of variation compares unfavorably with that obtained by the ACl-ASCE Committee 326 equation for the same test beams.

It must be remembered that the values obtained for $V/bd\sqrt{f'c}$ according to Equation 6.4 are highly dependent upon the values calculated for crack height and width as outlined in Appendix A. It is interesting to note that a better fitting equation for the test beams (Figure 6.4) might be of the form:

$$\frac{V}{bd\sqrt{f'c}} = C_1 + C_2\frac{\Delta X}{S} \qquad (6.5)$$

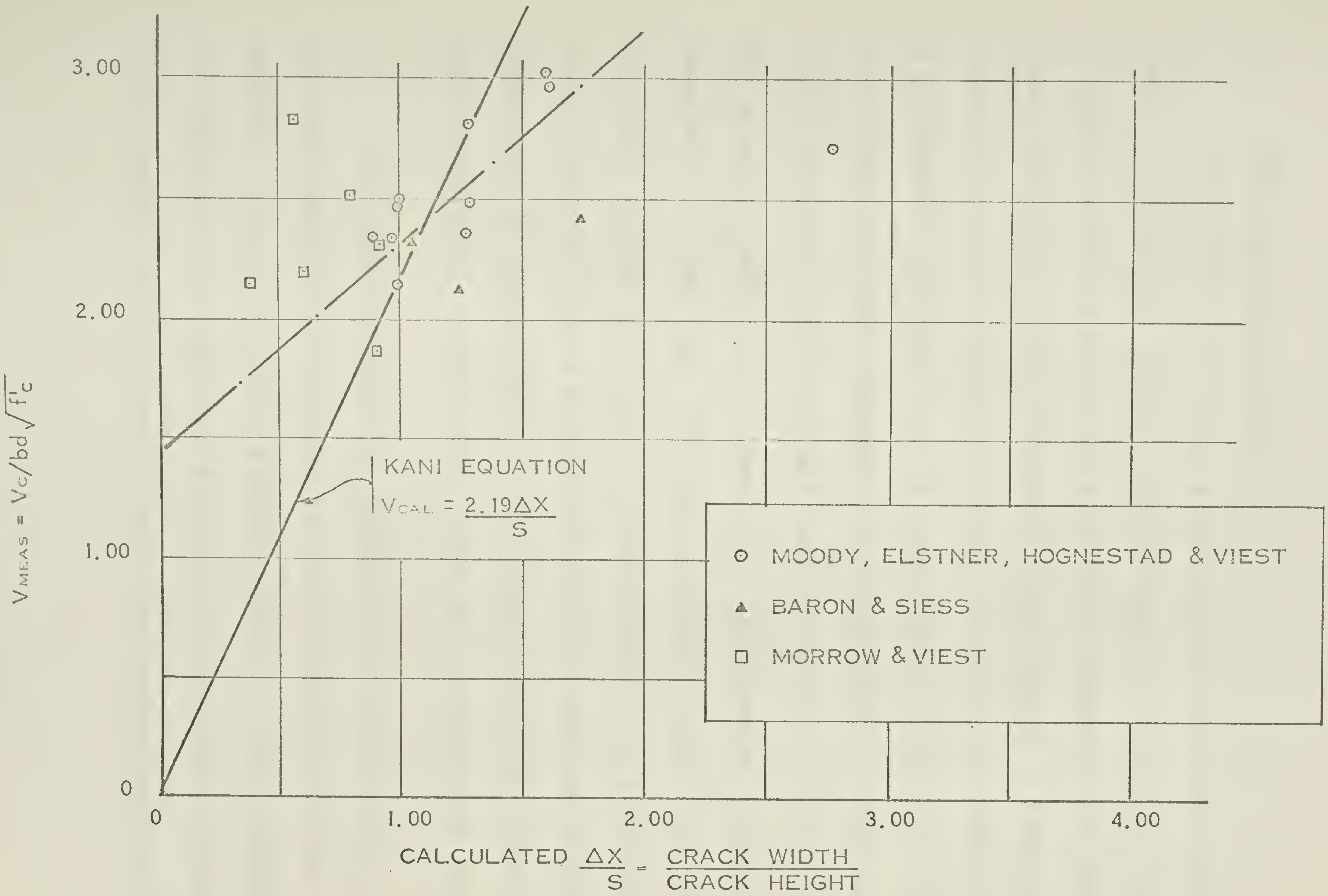


FIGURE 6.4 COMPARISON OF MEASURED SHEAR TO CRACK FACTOR

6.5 Flexure-Shear Theory

In the case of simply supported prestressed concrete beams, the research done at the University of Illinois showed a good correlation between the shear corresponding to flexure-shear cracking and the calculated shear causing a flexural crack at distances varying from d/2 to d from the load point. A prestressed concrete beam is essentially a reinforced concrete beam except for the difference in longitudinal reinforcement. If the concrete properties and overall geometry are the same for a reinforced concrete beam and a "prestressed concrete" beam with no prestress, the shear strength or inclined cracking loads of the two should be nearly equal. It should be noted, however, that a reinforced concrete beam has a higher percentage of longitudinal reinforcement which has greater bonding surface, a lower yield point and no prestress force.

The writer applied the prestressed flexure-shear cracking load Equation 5.37 to the ACI-ASCE Committee 326 test results. In applying this equation, however, the constants 0.6 and 0.5 in the right hand portion of Equation 5.36 were re-determined so as to give a least mean squared deviation.

The revised equation was as follows:

$$\frac{Vcr}{b'd\sqrt{f'c}} = 2.5 + \frac{Mcr}{(M/Vd-0.5)b'd^2\sqrt{f'_c}} \qquad (6.6)$$

Where:

M_{cr} = calculated flexural cracking moment for concrete.

The large increase in the constant from 0.6 to 2.5 may be attributed to:

(1) Increased dowel resistance, (to be discussed later).

(2) The fact that Equation 5.37 was developed as a lower bound solution whereas Equation 6.6 represents a mean value solution.

(3) The relative effect the concrete tensile strength has on the initial flexural crack.

The effect of inelastic concrete tension stresses are much less in prestressed beams. In the case of a prestressed concrete beam the concrete has a compressive prestrain due to the effective prestress which reduces the percent effect of the tensile strength of the concrete in relation to the cracking load. This should also reduce the scatter of test results due to the wide

variation in concrete strengths.

The ratio of measured to calculated cracking shears and the coefficient of variation are given in Table 6.1, although the coefficient of variation was fair it did not improve on that obtained by ACl-ASCE Committee 326.

Further study showed that the ratio of measured shear to calculated shear from Equation 6.6 did not appear to be affected by changing concrete strength, or width to depth ratio. It was affected, however, by changes in the ratio of steel, p = As/bd as shown in Figure 6.5. Use of a transformed section in the calculation of M_{cr} in Equation 6.6 did improve the results very slightly.

The major apparent physical differences, between the longitudinal strand reinforcement of a prestressed beam and the deformed bars of a reinforced concrete beam are the size, amount and ability to develop bond. These differences may result in:

(a) Significant shear transfer by doweling forces in a reinforced concrete beam.

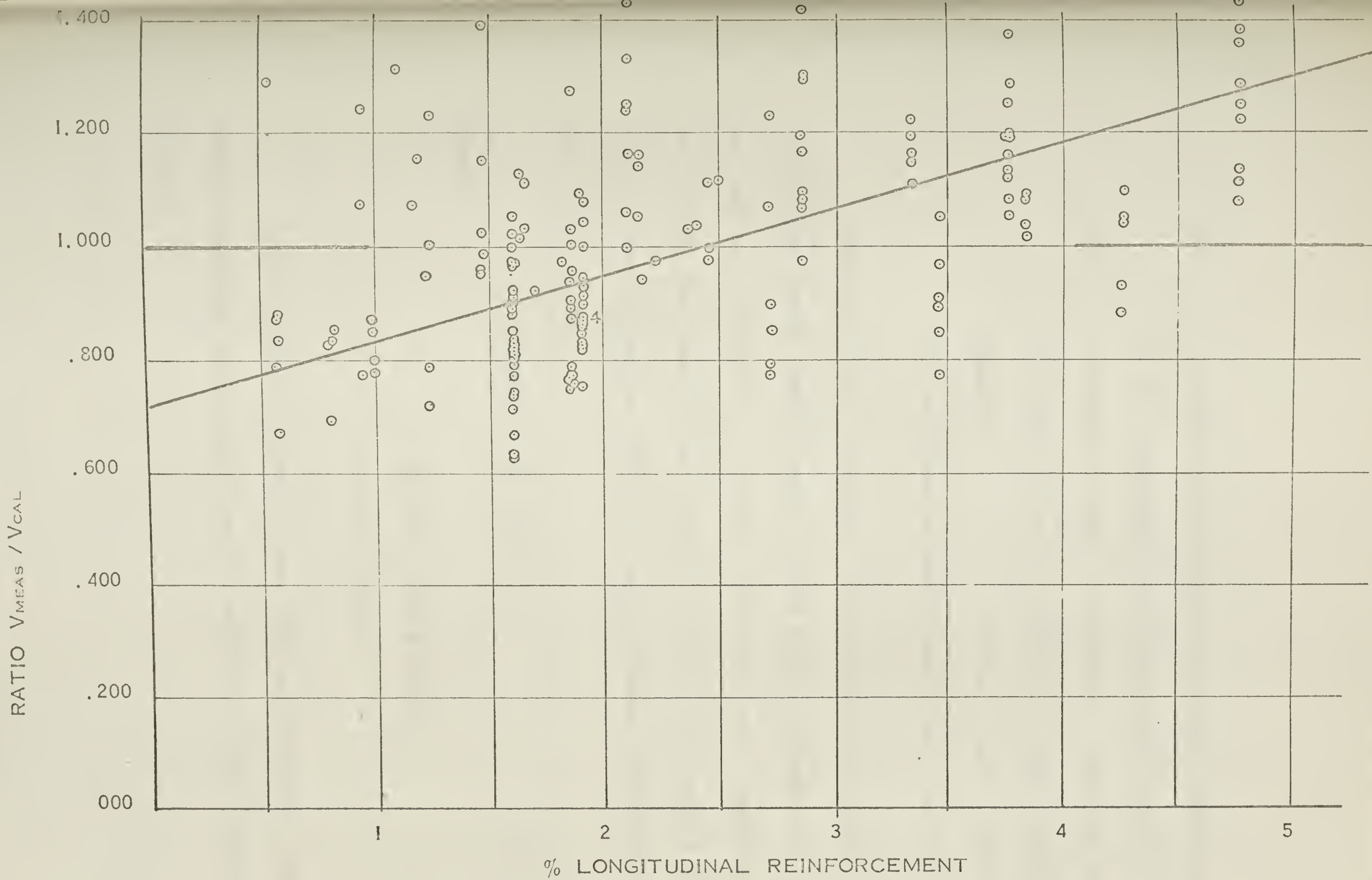


FIGURE 6.5 COMPARISON OF MEASURED SHEAR TO PERCENT LONGITUDINAL REINFORCEMENT

(b) A reduced crack width and crack spacing because of the increased bond and steel area.

(c) A reduced height of flexural cracking as would be predicted by Equation 6.2, developed in Appendix "A" to relate the crack height to the concrete strength and steel percentage.

Accepting the conclusion that longitudinal bars in reinforced beams resist part of the shear, Equation 6.6 was revised in an effort to account for doweling. Based on the tests and the theoretical analysis given in a report by Krefeld and Thurston (1962) the approximate doweling shear was obtained as:

$$V_{DOWEL} = \frac{c}{d}(1+3.2p) \qquad (6.7)$$

Where:

$p \quad = A_s/bd$ = steel ratio

$c \quad$ = cover = h-d

The following equation was obtained for the flexure-shear cracking load including the effects of the doweling action.

$$\frac{V_{cr}}{b'd\sqrt{f'c}} = 1.16 + \frac{M_{cr}}{(M/Vd-0.5)b'd\sqrt{f'_c}} + 4.35\frac{c}{d}(1+3.2p) \qquad (6.8)$$

The results obtained using Equation 6.8 were only slightly better than those of Equation 6.6 but still higher than those obtained by ACl-ASCE Committee 326.

It is interesting to compare the mean intercept value for the Sozen and Hawkins data in Figure 5.7 with the constant, 1.16 in Equation 6.8. The intercept on the vertical axis of a best fit line in Figure 5.7 is approximately 0.95 and compares reasonably with the constant 1.16 in Equation 6.8. The addition of a doweling term measured by the variable p appears to indicate that the steel ratio p is much more of a direct factor in determining shear strength in simple reinforced beams.

Equation 6.8 does not attempt to take the effect of the height of flexural cracking into account.

6.6 Summary

Due to the fact that certain terms or variables used in the different equations were neither given nor directly

obtainable from ACl-ASCE Committee 326 test data, empirical formulas were used to compute the unknown values. Therefore, the results of the separate investigations of each previously discussed theory are more of a qualitative nature than quantitative and numerical comparisons will not be presented in this summary. The following summary table was developed from the preceeding investigations and studies.

TABLE 6.3

BASIC FACTORS CONSIDERED PERTINENT IN THE DEVELOPMENT OF THEORIES

Theory and Equation

Factor or Parameter	1956 ACl Code Eq. 5.6	Guralnick Eq. 5.8	Van den Berg Eq. 5.20	1963 ACl Code Eq. 5.24	Kani Eq. 5.30	Flexure-Shear Eq. 5.37
(1) a/d ratio or M/Vd ratio	No	No	Yes	Yes	Yes	Yes
(2) Concrete Properties	Yes	Yes	Yes	Yes	Yes	Yes
(3) Percent tensile Reinforcement	No	No	Yes	Yes	Indirectly	Indirectly
(4) Beam Size and Cross section	Yes	Yes	Yes	Yes	Yes	Yes
(5) Method of Loading	No	No	Could be considered with different Eq	Yes		
(6) Creep and Shrinkage	No	No	No	No	No	No

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 General

Although recent extensive experimental and analytical research have greatly increased the degree of understanding and knowledge of the behavior of reinforced concrete beams under shear and diagonal tension stresses, research has failed to determine the distributions of shear and flexural stress over a cross section. A realization that the combination of shear and flexural stress in a cracked reinforced concrete beam is a complex problem involving many variables makes it evident that the diagonal tension stresses are generally indeterminate. Add to this the indeterminant stress problem resulting from internal stresses produced by such factors as shrinkage, creep, rate of loading, temperature and other volume changes and it becomes obvious why investigators executing shear projects have failed to provide a completely rational relationship for diagonal tension cracking. For this reason,

the completely rational procedure has been abandoned in favor of a semi-empirical approach which takes into account the variables which the many hundreds of test results have shown to affect the diagonal tension strength.

7.2 Conclusions

The following are the conclusions drawn from the study of the comparisons of the different theories and the variables affecting diagonal tension strength in reinforced concrete beams without web reinforcement.

(a) With Respect to a Design Equation

(1) Of the six methods statistically studied, the ACl-ASCE Committee 326 Equation 5.22 gave the best general solution for the range of values analyzed.

(2) The flexure-shear expression, Equation 6.8 gave a reasonable solution without applying any limits.

(3) The classical shear Equation (1956 ACl Code Equation, Eq. 5.6) and the expression developed by Guralnick (1960) consider concrete strength as the only direct variable and thus should not be used.

(4) The expressions presented by Kani (1964) could not be justified as the theory was developed around the parameter, X/S, which was not defined by Kani. Although the writer developed a simplified equation for determining X/S; no testing programs were conducted to substantiate the solution.

(b) <u>With Respect to Variables Affecting Inclined Cracking</u>

(1) The inclined cracking or diagonal cracking load tends to have ranges wherein the affect of variables change depending on the mode of failure.

(2) There is a marked influence on the inclined cracking load due to:

(i) The percentage of longitudinal reinforcement p,

(ii) The quality of concrete as expressed by the tensile strength f't, and

(iii) By the dimensionless quantity a/d or M/Vd, depending on the type of loading.

(3) Statistically derived semi-rational expressions appear to give better correlation as they partly account for such unknown variables as shrinkage and creep.

(4) The effects of member size, shape and type of loading on shear strength of members have not been properly evaluated to date and are generally skirted by most shear equations.

7.3 Recommendations:

(1) The somewhat opposing thoughts with respect to the interaction of bond on inclined cracking and shear capacity should be verified by a test program.

(2) The effect of flexural cracking on the inclined cracking load of beams with varying a/d ratios should be further studied in order to substantiate the flexure-shear approach to inclined cracking.

(3) Further tests for a range of variables using restrained beams with applied end moments would be desirable to substantiate Van den Berg's conclusion that there was a linear relationship between the applied pure shear force and the shearing stress.

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APPENDIX A

"DEVELOPMENT OF CRACK FACTOR"

APPENDIX A

DETERMINATION OF CRACK FACTOR, X/S

A.1 General:

It must first of all be emphasized that cracking is essentially a phenomenon characterized by uncertainty. Cracking depends fundamentally on actual tensile strength of concrete at each point of the member under consideration. It is well known that tensile tests give only a single minimum value for that strength, for which the statistical laws of variation are not known. It is therefore quite pointless to hope for a formulae possessing a high degree of accuracy. The simplest formula would therefore probably be the best.

A.2 Crack Spacing "$\triangle X$"

The expression developed by M. Bruce (1960) denotes the average crack spacing $\triangle X$, by the simple formula:

$$\triangle X \text{ Average} = \frac{3}{2Nd} \qquad (A.1)$$

Where:

Nd = A specific coefficient relating the cross sectional shape of the reinforcing bars. ($Nd = 1.6$ for deformed bars).

$$t = \frac{\text{Area of concrete around bars}}{\text{Perimeter of Bars}}$$

Therefore:

$$t = b \times 2(h-d) / \Sigma \odot \qquad \text{(a.2)}$$

The expression used was therefore given by:

$$\Delta X \text{ average} = 0.9375.b \; \frac{(2(h-d))}{\Sigma \odot} \qquad \text{(A.3)}$$

Where:

b, d, h are all dimensions of the cross section

$\Sigma \odot$ = sum of the reinforcing bar perimeters.

The validity of equation A3 was somewhat checked by comparing its results with those obtained by scaling off values from test photos.

A.3 Crack Height "S"

It was assumed that the crack height due to flexure was a function of:

(i) Tension reinforcing percentage, P, and

(ii) Concrete tensil strength. f'_t.

Based on the concrete properties assumed in Chapter 4 and a linear strain distribution it was possible to derive moment-crack height curves Figure (3.2) by a trial and error solution. By arbitrarily defining the crack height, S, as the height at 1.1 times the flexural cracking moment, enabled an independent plot of the variables f'_c and p against crack height, S.

The relationships of Figures A.1 and A.2 were incorporated to give Figure A.3. By limiting the ratio of $f'_{c/p}$ where f'_c is in kips per square inch and P in percent, the following expression was derived for maximum crack height, S:

$$S = \left(A + \frac{C\, f'_c}{p}\right) d \qquad \text{(A.4)}$$

Where:

d = effective depth

A = constant = -.08

C = constant = 0.126 in in^2/kip

f'_c = compressive strength in $kips/in^2$

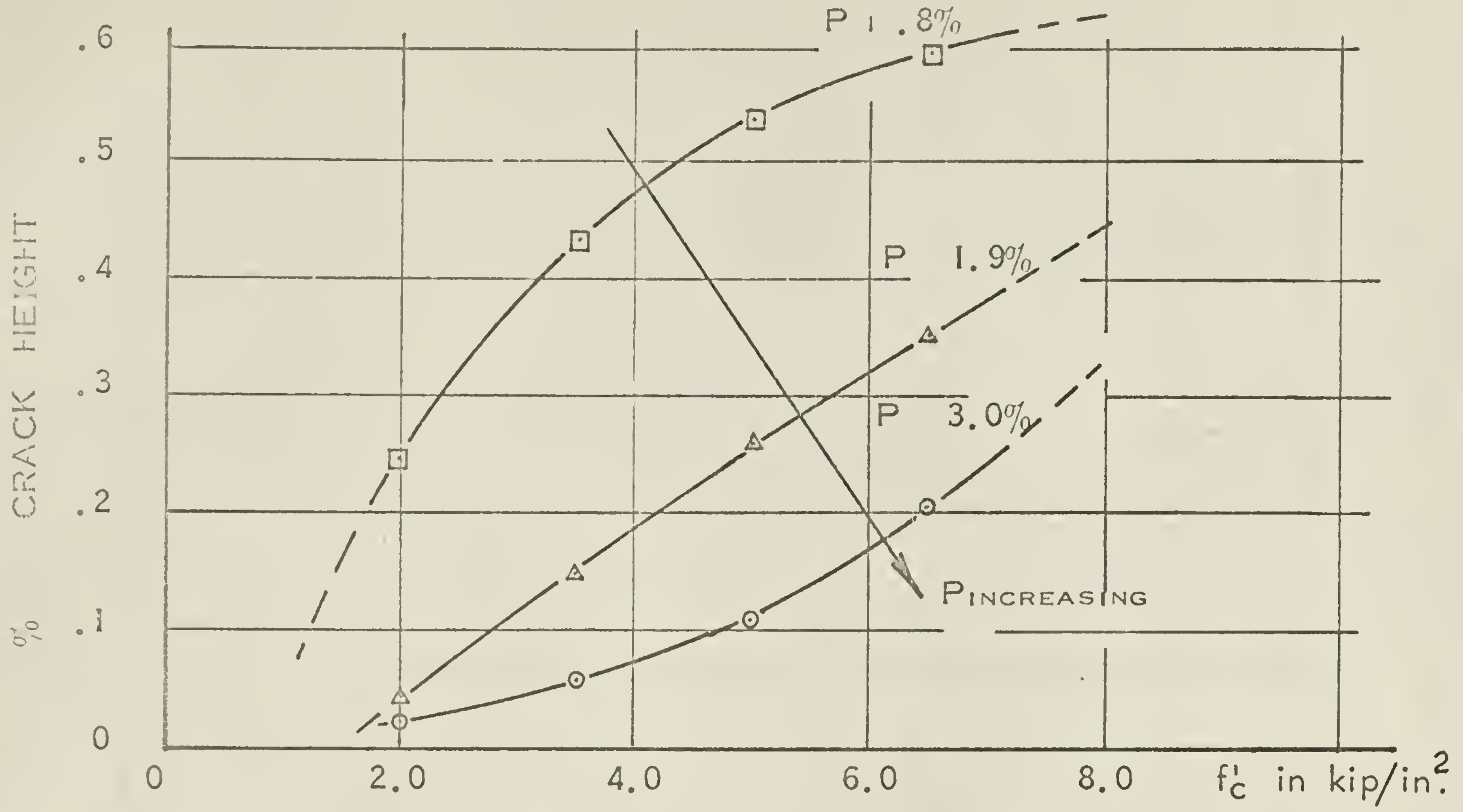

FIGURE A.2 COMPARISON OF CRACK HEIGHT TO CONCRETE COMPRESSIVE STRENGTH

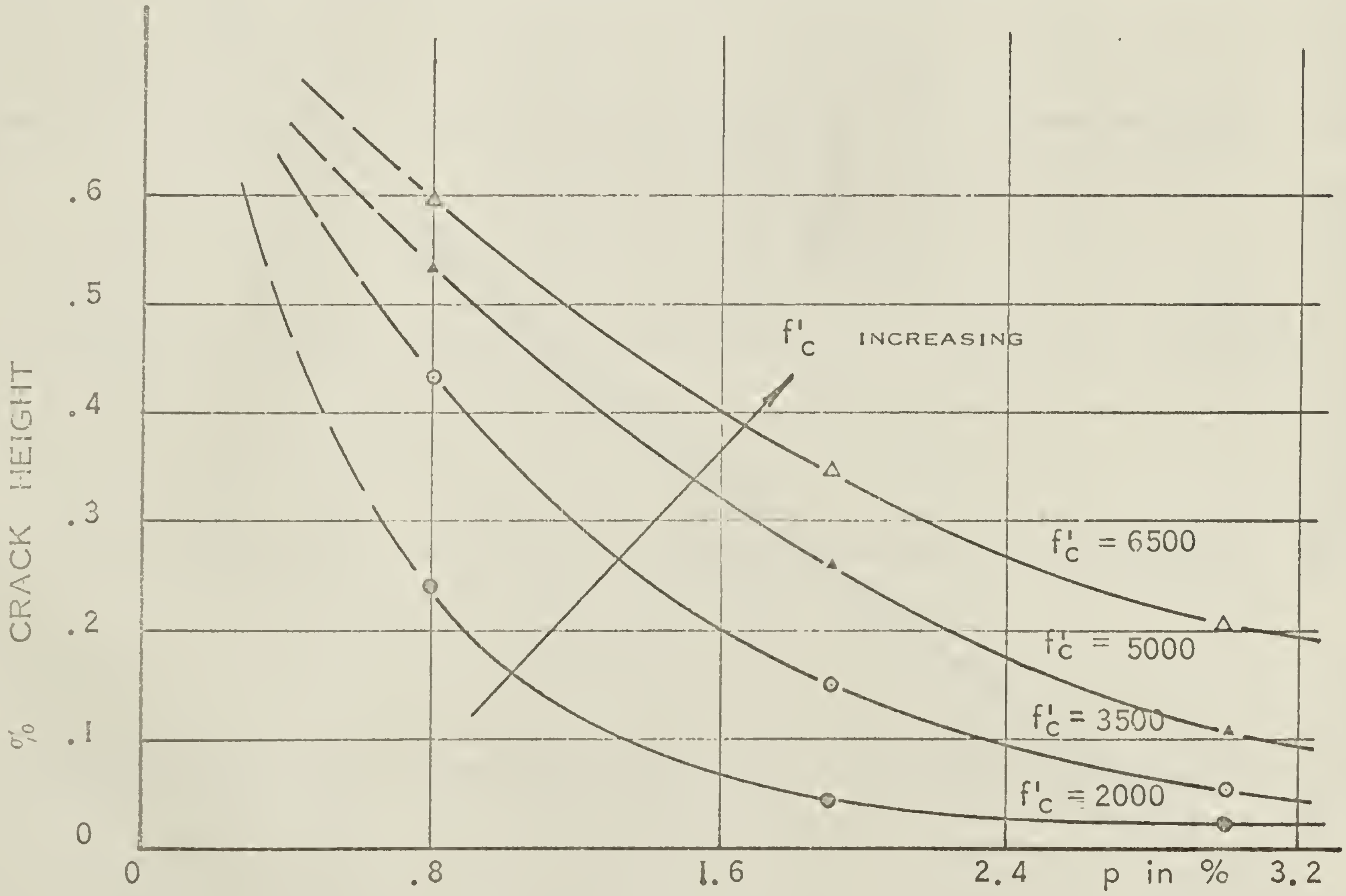

FIGURE A.1 COMPARISON OF CRACK HEIGHT TO PERCENT LONGITUDINAL REINFORCEMENT

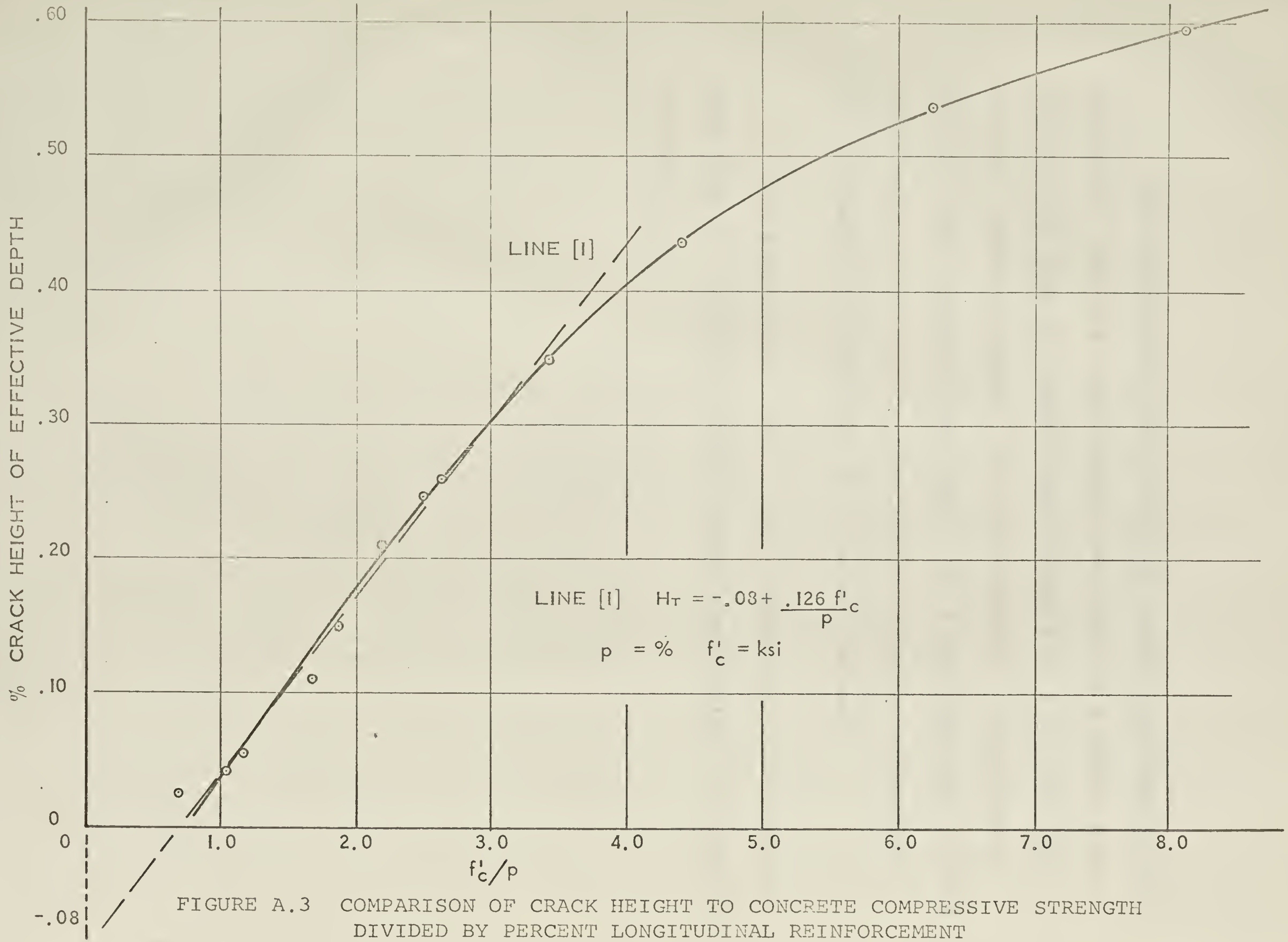

FIGURE A.3 COMPARISON OF CRACK HEIGHT TO CONCRETE COMPRESSIVE STRENGTH DIVIDED BY PERCENT LONGITUDINAL REINFORCEMENT

The relationship for the crack height was based on the assumption that the crack height had risen to a stable position by the time the moment had increased to 1.1 times the flexural cracking moment. For the values of f'c/p greater than 2.0 this appears to be true. For f'c/p less than 2.0 the crack height continues to rise significantly as the moment is increased and Expression (A.4) cannot be used.

The Expression (A.4) is also limited to values of f'c/p less than 4.0, so as to remain within the straight line portion of Figure A.3.

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